

The Multi-Model Partitioning Theory: Current Trends and Selected Applications

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Abstract: The multi-model partitioning approach to adaptive estimation and control was introduced by Lainiotis forty years ago. Since then, three generations of multi-model partitioning algorithms have appeared and numerous applications of the multi-model partitioning approach have been developed. In this paper, a concise review of the theory underlying the multi-model partitioning approach is presented, as well as a brief survey of selected applications of the approach.

Key words: Multi-Model Partitioning, Adaptive Lainiotis Filter, Estimation, Identification, Extended Kalman Filter

1. INTRODUCTION

The problem of estimating the parameters and/or states of a nonlinear system is a truly complicated one. The majority of realistic systems is either by nature nonlinear or has nonlinear observation structures or is partially unknown. Moreover, optimal nonlinear filtering is far less precise than its linear counterpart. In general, one has to accept that, when dealing with nonlinear dynamical system estimation, an analytical solution in closed form is not likely to be available, and instead, computational algorithms should be sought in their place.

The difficulties in implementing an optimal solution for the general nonlinear problem, led to the search for alternative, suboptimal configurations. Such approximations include the well known Kalman filter in its extended (nonlinear) forms ¹, as well as, the various implementations

of the Multi-Model Partitioning (MMP) theory introduced by Lainiotis²⁻⁵ almost four decades ago.

The Multi-Model Partitioning approach has received a great deal of attention due to its success in decomposing complex problems into simpler sub-problems, and in handling effectively structural or parametric uncertainties. Since the introduction of the multi-model partitioning approach, three generations of multi-model partitioning algorithms have appeared⁶ and numerous applications of the multi-model partitioning approach have been developed. In this paper, a review of the theory underlying the MMP approach is presented, as well as a brief survey of selected applications of the approach.

2. LAINIOTIS MULTI-MODEL PARTITIONING THEORY

2.1 The Dynamical Model

The general state space model for a discrete time nonlinear stochastic system with additive Gaussian excitation and measurement noise has the following form:

$$x(k+1) = f(k, x(k)) + g(k, x(k)) w(k) \quad (1)$$

$$z(k) = h(k, x(k)) + v(k) \quad (2)$$

where, $f(\cdot)$, $g(\cdot)$ and $h(\cdot)$ are nonlinear functions of the state that depend on the index k , $w(k)$ and $v(k)$ are zero mean, Gaussian noises having variances $Q(k)$ and $R(k)$ respectively. The initial state of the system, $x(0)$, is assumed to be described by a known probability density function $p(x(0))$.

The objective of the optimal estimator⁷⁻⁸ is to obtain the optimal estimates of the stochastically varying state vectors $X_k = \{x(1), x(2), \dots, x(k)\}$, given the available information contained in the related sequence of measurement vectors $Z_k = \{z(1), z(2), \dots, z(k)\}$.

The design of optimal nonlinear estimators may seem promising, however, the probability densities involved, are not Gaussian and, as a result, they cannot be completely described from the first two moments thus, its functional recursive formulation is impractical for real nonlinear estimation problems⁹. On the other hand, if the system is linear and the disturbances are assumed Gaussian, only a finite statistic is sufficient, consisting of the state's mean and error covariance. The linear Kalman¹⁰, the linear Lainiotis¹¹, or any other linear optimal filter can provide the means for a recursive update

of such a sufficient statistic. A widespread method for simple nonlinear problems is the extended Kalman filter (EKF)¹. In most common EKF implementations the nonlinear functions are linearized, via a first-order Taylor series expansion at the most recent state estimate.

Partially unknown, complex or highly nonlinear problems can be decomposed into simpler sub-problems (multiple models - MM) that can be solved by the aforementioned linear or extended filters. The same approach is applied either for uncertainties or nonlinearities and results to a set of candidate models that span the model space of the problem. Each model is a partial realization of the problem and it is characterized by a parameter set θ with an *a-priori* probability density function $p(\theta)$. Vector θ contains the unknown parameters and spans a usually discrete space Ω_θ , either because θ is naturally discrete or because it has been discretised. The discrete probability density function becomes $p(\theta_i)$, $i = 1, \dots, S$ where S is the number of possible instances of θ in Ω_θ .

This nonlinear partially unknown state space model has the following form:

$$x(k+1) = f(k, x(k); \theta) + G(k; \theta) w(k) \quad (3)$$

$$z(k) = h(k, x(k); \theta) + v(k) \quad (4)$$

and, the linear partially unknown state space model will be:

$$x(k+1) = F(k+1, k; \theta) x(k) + G(k; \theta) w(k) \quad (5)$$

$$z(k) = H(k; \theta) x(k) + v(k) \quad (6)$$

where, $F(\cdot)$, and $H(\cdot)$ are the (possibly unknown) transition and observation matrices respectively. The unknown parameters are denoted by the vector θ , which, if known, would completely specify the model. $w(k)$ and $v(k)$ are uncorrelated zero mean, Gaussian noises having variances $Q(k; \theta)$ and $R(k; \theta)$ respectively. The initial state vector $x(0)$, is conditionally Gaussian for given θ with mean $x(0/0; \theta)$ and variance $p(0/0; \theta)$ uncorrelated to $w(k)$ and $v(k)$.

2.2 The Multi-Model Partitioning Filter

The problem is to estimate the system state at time k , given measurements of the observation vector up to and including time k . This estimate is denoted by $\hat{x}(k/k)$ and the estimation error covariance is denoted by $P(k/k)$. The solution to this problem is given in by Lainiotis⁴ and its Multi-Model Partitioning approach, as follows:

Given the observation sequence Z_k , the Minimum Mean Square Estimate (MMSE) $\hat{x}(k/k)$ of $x(k)$ is given by:

$$\hat{x}(k/k) = \sum_{i=1}^M \hat{x}(k/k; \theta_i) p(\theta_i/k) \quad (7)$$

where, $\hat{x}(k/k) = E[x(k)/Z_k]$ and, $\hat{x}(k/k; \theta_i) = E[x(k)/Z_k; \theta_i]$. The *a posteriori* probability of θ assuming the value θ_i given Z_k , $p(\theta_i/Z_k) = p(\theta_i/k)$, is given by:

$$p(\theta_i/k) = \frac{L(k/k; \theta_i) p(\theta_i/k-1)}{\sum_{i=1}^M L(k/k; \theta_i) p(\theta_i/k-1)} \quad (8)$$

where,

$$L(k/k; \theta_i) = |P_z(k/k-1; \theta_i)|^{-1/2} \exp \left\{ -\frac{1}{2} \tilde{z}^T(k/k-1; \theta_i) P_z^{-1}(k/k-1; \theta_i) \tilde{z}(k/k-1; \theta_i) \right\} \quad (9)$$

Here the innovation process $\tilde{z}(k/k-1; \theta_i)$ is a zero-mean white process with covariance matrix $P_z(k/k-1; \theta_i)$ obtained by a conditional filter (e.g., Kalman) matched to the system model with parameter θ_i :

$$\tilde{z}(k/k-1; \theta_i) = z(k) - H(k; \theta_i) \hat{x}(k/k-1; \theta_i) \quad (10)$$

$$P_z(k/k-1; \theta_i) = H(k; \theta_i) P(k/k-1; \theta_i) H^T(k; \theta_i) + R(k; \theta_i) \quad (11)$$

The estimation error covariance matrix $P(k/k)$ is given by:

$$P(k/k) = \sum_{i=1}^S \left\{ P(k/k; \theta_i) + [\hat{x}(k/k; \theta_i) - \hat{x}(k/k)] [\hat{x}(k/k; \theta_i) - \hat{x}(k/k)]^T \right\} p(\theta_i/k) \quad (12)$$

where, $P(k/k; \theta_i) = E \left\{ [x(k) - \hat{x}(k/k; \theta_i)] [x(k) - \hat{x}(k/k; \theta_i)]^T / Z_k; \theta_i \right\}$.

Again, the conditional estimates $\hat{x}(k/k; \theta_i)$, $\hat{x}(k/k-1; \theta_i)$ and the associated error covariance matrices $P(k/k; \theta_i)$, $P(k/k-1; \theta_i)$, are obtained by a conditional filter (e.g., Kalman) matched to the system model with parameter θ_i .

The Multi Model Partitioning Filter (MMPF) structure that implements the above solution is shown, in a block diagram form, in figure 1.

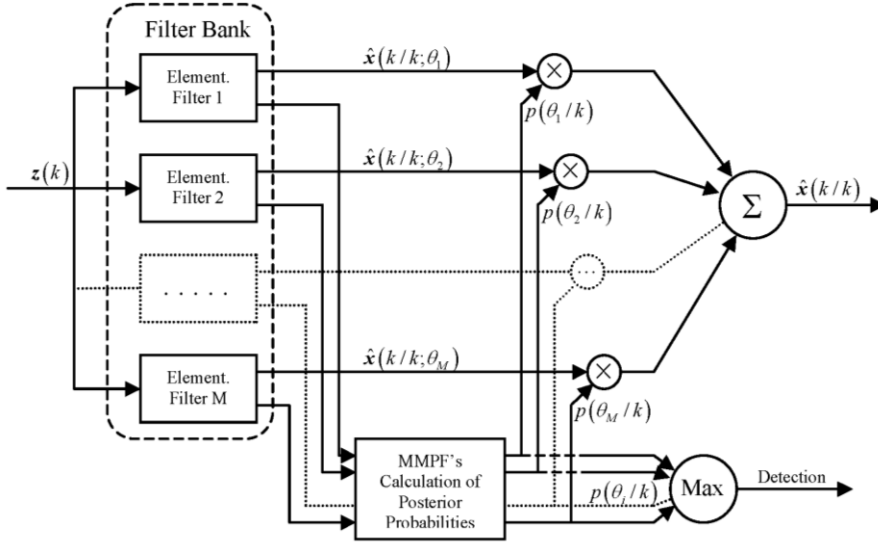


Figure 1. Block diagram of the Multi-Model Partitioning filter implementation.

Comment 1: It is not necessary to use Kalman filters as the model-conditional filters of the MMPF; any other filter can be employed instead (e.g., Extended Kalman Filter, Lainiotis per step partitioning filter, or even another MMPF) as long as it provides the quantities required by the MMPF. Thus, the MMPF can handle a nonlinear model by employing a nonlinear filter as its elemental filter.

Comment 2: The overall estimate of the MMPF can be taken either to be the individual estimate of the elemental filter exhibiting the highest posterior probability (called “MAP – Maximum A Posteriori – estimate”) or the weighted average of the estimates of the elemental filters, where the weights are simply the posterior probabilities associated with each estimate (called “MMSE – Minimum Mean Square – estimate”)¹²⁻¹³.

Comment 3: In real life problems, where the unknown parameter usually spans a continuous space or follows a probabilistic distribution function, one must somehow discretize the space, choosing a finite subset of parameters such that it can, in some sense, efficiently “cover” the space. Several such discretization strategies have been proposed at times. It is intuitively clear that the more terms the finite sum has, the better the approximation. The issue of “how good” a finite covering of an infinite space is in the context of multiple model estimation and control is discussed in detail^{1,14,15}.

Comment 4: It should be evident that the adaptive estimator inherently serves also the purpose of identification of the unknown parameter. When the true parameter value lies within the assumed sample space (hence one of

the MMPF assumed models is the true one), the estimator converges to this value. When the true parameter does not belong to the assumed sample space, the estimator converges to that value in the sample space that is closest (the one that minimizes the Kullback information measure) to the true one^{1,35}.

2.3 Evolution of the MMP Algorithms

The MMPF presented in 2.3 represents the **first generation** of Multi-Model Partitioning algorithms. These algorithms ask all individual sub-filters to perform simultaneously and they produce the overall output based on their performance. The *a posteriori* probability $p(\theta_i/k)$ for a well performing sub-filter increases, while a poorly performing or deviating sub-filter is finally ignored as its probability approaches zero.

The superiority of a Multi-Model Partitioning algorithm over a non-MM algorithm stems from its flexibility. The MMP algorithm decides and selects the best elemental filter *a posteriori*, while any single model algorithm must select its “best” filter *a priori*.

The main characteristic of the first MMP generation is that the elemental filters work independently. They are also called Autonomous MMP algorithms.

The **second generation** of MMP focuses on internal cooperation between the elemental filters and is best represented by the Interacting Multi-Model (IMM) algorithm¹⁷. Another implementation is the Per-Sample Initialized Adaptive Lainiotis Filter (PSI-ALF) where the “best” estimator communicates its results to the other candidates in the filter bank¹⁸.

The second generation outperforms the algorithms of the first generation by interacting and communicating. In the effort for improved performance, the first two generations of MMP algorithms are based on a given model set and they try to improve the MMP algorithm as well as the elemental filtering algorithms.

The second MMP generation focuses on interaction and collaboration between the elemental filters and with the MM estimator. They are also called Cooperative MMP algorithms.

A **third generation** of MMP algorithms, currently receiving great attention, attempts to improve the performance of the Multi-Model Partitioning approach by designing a better model set¹⁹⁻²⁰. The structure of the filter bank now becomes variable; its members may be added or removed and various criteria and measures for the design, comparison, and choice of the model-set were proposed²¹.

Model-set design can be formulated as that of finding the optimal model set given data based on optimization techniques. Such an algorithm was

proposed¹³ based on a genetic algorithm (GA) combined with the Lainiotis MMPF. The initial population of the model set evolves in time according to the rule of survival of the fittest. New members are created using the Reproduction operator, Crossover operator and Mutation operator. The merging of the two techniques led to the development of an algorithm, capable of effectively dealing with situations where the unknown parameter space is of infinite cardinality²².

The third MMP generation focuses on the improvement of the model set and they are usually called Variable-Structure MMP algorithms.

3. SELECTED APPLICATONS OF LAINIOTIS MULTI-MODEL PARTITIONING THEORY

3.1 Target Tracking

Multi-Model Partitioning (MMP) methods have been generally considered the main approach to maneuvering target tracking under parametric or structural model uncertainty and nonlinearity. MMP methods incorporating nonlinear filters are clearly the most promising tool for the target tracking problem²³.

3.1.1 Radar Target Tracking

Radar target tracking has been an area of intensive research for many years. Traditionally, the problem consists of tracking a target utilizing noisy position measurements available to the radar tracking station (figure 2).

A typical nonlinear target-radar model in state-space form requires, A) For the target state: a 1 to 3-dimensional equation (i.e. position, velocity, acceleration) for each Cartesian coordinate (X, Y, Z, not being always decoupled), and, B) For the radar observations: a 1 or 2-dimensional equation (i.e. the observed value and its rate) for each Spherical coordinate (range, bearing, elevation). For example, for a target Cartesian coordinate, e.g. X, the 2-D state equation may be of the form:

$$\begin{bmatrix} x(k+1) \\ \dot{x}(k+1) \end{bmatrix} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x(k) \\ \dot{x}(k) \end{bmatrix} + \begin{bmatrix} T/2 \\ 1 \end{bmatrix} W_v(k) \quad (13)$$

and, for a radar spherical coordinate, e.g. the range, the 2-D observation equation may be of the form:

$$\begin{bmatrix} r(k) \\ \dot{r}(k) \end{bmatrix} = \begin{bmatrix} \sqrt{x(k)^2 + y(k)^2 + z(k)^2} \\ \frac{x(k)\dot{x}(k) + y(k)\dot{y}(k) + z(k)\dot{z}(k)}{r(k)} \end{bmatrix} + \begin{bmatrix} V_{r_v}(k) \\ V_r(k) \end{bmatrix} \quad (14)$$

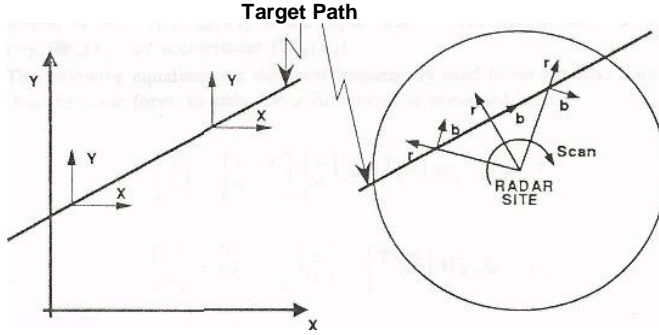


Figure 2. Cartesian (a) and Spherical (b) coordinate systems for radar target tracking.

Early attacks to the problem assumed straight and level path of the target and proposed simple, constant gain filtering algorithms, operating on linear models, for its solution. This early approach includes the most common α - β and α - β - γ filters, as well as their numerous derivatives. This approach, naturally, fails when the target is in fact maneuvering (thus introducing uncertainty and nonlinearity), which it is likely to do in a real world environment. Realization of this fact led to the development of more sophisticated (nonlinear) models and algorithms.

Among the state-space approaches, two classes stand out: the ones based on the Extended Kalman Filter (EKF) that has been known to suffer from numerical instabilities and from the so-called divergence phenomenon, and the ones based on the Multi-Model Partitioning (MMP) approach. The MMP algorithms have been shown, in several comparative studies^{25,26} to perform better than the EKF-based ones.

The class of algorithms based on the Multi-Model Partitioning approach comprises, among others, the work by Moose²⁴ and by Blom and Bar-Shalom¹⁷. Tracking algorithms based on the Multi-Model Partitioning approach were also designed for the special case of collaborating targets such as civilian aircrafts^{27,18}. These tracking filters receive Aircraft Derived Data (ADD) measurements of heading, through the secondary surveillance radars of Air Traffic Control systems. Again all MMP algorithms, the adaptive Lainiotis filter, the Per Sample Initialized ALF (figure 3) and the

Multi-Model Input estimation filter, performed better than the EKF especially in the case of maneuvering targets.

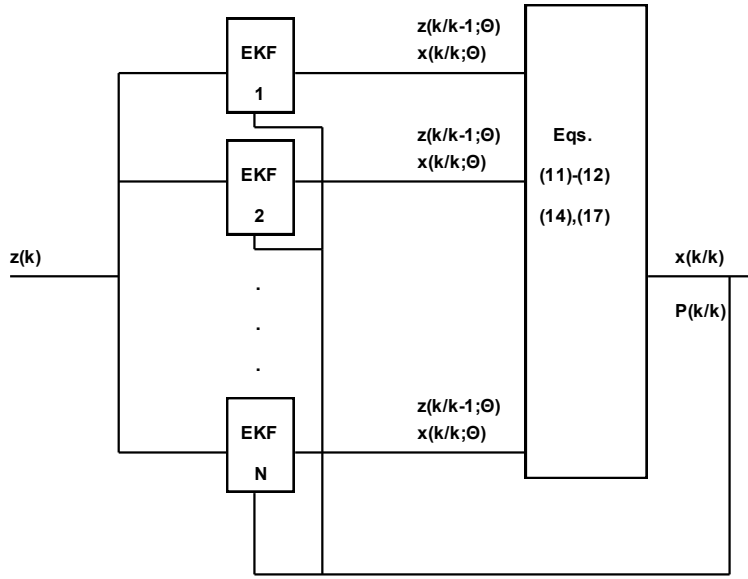


Figure 3. Block diagram of the Per Sample Initialized Adaptive Lainiotis Filter, based on its Multi-Model Partitioning approach¹⁸.

However, there is a price to pay for this improved performance of all Multi-Model Partitioning algorithms: increased computational requirements. Indeed, as MMP filters (ALF, PSIALF, MMIE, etc.) utilize a bank of sub-filters (Kalman, EKF, etc.) instead of a single filter, their requirements are several times higher. Nevertheless, as already stated, it is possible to implement the bank of sub-filters in parallel, thus eliminating the effect of any computational overhead to the speed of the MMP algorithm²⁸.

3.1.2 Underwater and Passive Target Tracking

The problem of target tracking in the ocean environment is of great importance in military, oceanographic and fisheries applications. Because of the complexity of sound propagation in the ocean medium, the models describing the target observation process are nonlinear, and driven by non-Gaussian signals. Since no optimal solution can be obtained, research has focused in developing suboptimal, computationally efficient tracking algorithms. A major source of complexity is the possibility of target

maneuvering. Algorithms that do not adequately model possible target maneuvers are most likely to fail when the target does maneuver²⁹.

In the case of passive tracking of non-maneuvering targets, in Cartesian coordinates, the equations of motion of both the target and the observer yield the following state and measurement equations³⁰:

$$X(k+1) = X(k) + v_x T \quad (15)$$

$$Y(k+1) = Y(k) + v_y T \quad (16)$$

$$v_x(k+1) = v_x(k) \quad (17)$$

$$v_y(k+1) = v_y(k) \quad (18)$$

$$\beta(k) = \arctan \frac{Y(k) - Y_o(k)}{X(k) - X_o(k)} + e'(k) \quad (19)$$

where $X(k)$ and $Y(k)$ are the two components of the target position at time k , v_x and v_y are the target velocity components (assumed constant in time), $\beta(k)$ is the measured target bearing, $X_o(k)$ and $Y_o(k)$ are the observer position components at time k , T is the sampling interval and $e'(k)$ is a stochastic series of independent and identically distributed random variables with mean m and variance R . The state vector comprising X , Y , v_x , v_y is unobservable if the observer is stationary or moves continuously on a straight line. Therefore the observer is assumed to maneuver.

The application of the Multi-Model Partitioning approach to this problem is not straightforward. Indeed, observe from the model definition equations that the sample space of the unknown parameters is not naturally discrete. Therefore, some sort of discretization of the parameter space must be performed. The attempted approach is to divide the parameter space into M non-overlapping sub-areas. Over each sub-area, an independent filter, called a "subarea filter" is designed consisting of a bank of N Kalman filters and the MMPPF equations. It is clear that M such filters are required.

In the case of passive tracking of maneuvering targets, all models describing the observation process (i.e. the relative target-observer motion) are inherently nonlinear. Use of the modified polar set of coordinates³¹ has been shown to be beneficial over the use of the Cartesian coordinate system.

The problem of estimating the maneuvering target's relative position is shown to be equivalent to the problem of estimating an unknown and time-varying bias term of the plant noise process²⁹. Several algorithms have been reported in the literature, however, all these algorithms pertain to the case of time-invariant unknown bias, or at best to slowly time-varying bias; hence they are inappropriate for the case at hand.

In order to tackle the above problem an algorithm is developed^{32,33} capable of efficiently handling the problem of state estimation with time-

varying unknown bias of the plant noise process. The algorithm is derived by using the generic Multi-Model Partitioning approach to estimation and control, coupled with conventional constant bias estimation algorithms.

The proposed algorithm operates on a bank of elemental filters, each of which is conditioned on a change in bias taking place at a certain point of time. Two bias estimators, taken from the control literature, have been used as elemental filters in two different implementations of the proposed algorithm (figure 4).

The results show that the proposed algorithm performs very well even in adverse conditions, such as long range and high noise. Furthermore, it is able to detect fast enough and cope with abrupt, large-scale target maneuvers.

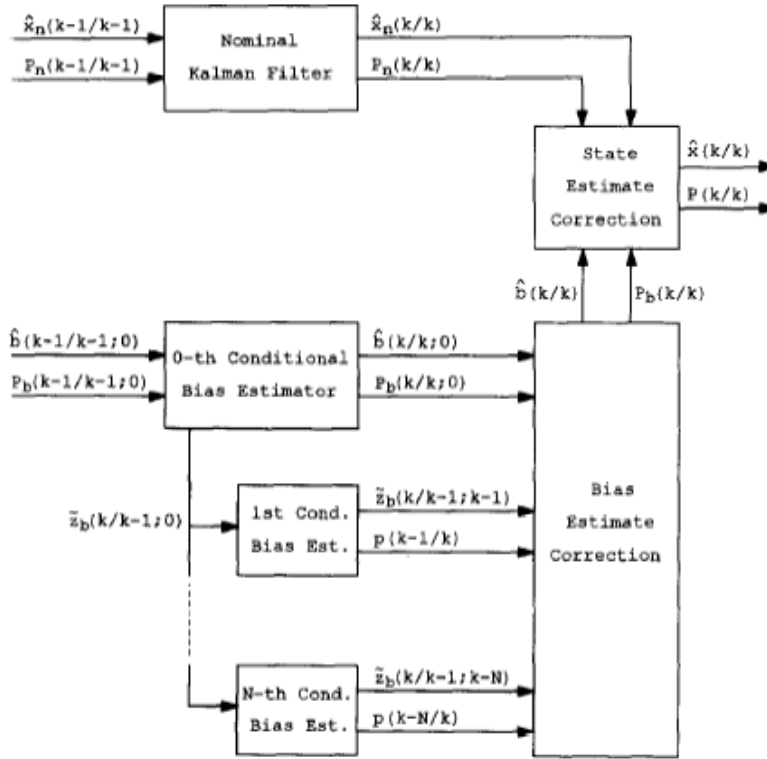


Figure 4: Block diagrams of the proposed Multi-Model Partitioned bias estimation algorithm.

Another problem closely related to passive target tracking is the problem of towed array shape estimation. This problem has received considerable attention in the past few years due to its importance in sonar and seismic applications³⁴. It is widely recognized that the shape of the towed array plays a critical role in the sophisticated processing techniques of hydrophone data.

In the past many methods have been proposed for estimating the shape of a towed array; such a method is using Kalman filters³⁵ for the task. The Kalman filter, at each instant in time, gives the least mean square estimate of the state of the system, when all the previous sampled time history of the measurement sensors is taken into account.

The approaches based on the Kalman filter do not usually perform adequately when facing a gross maneuver (such as a turn) or a perturbation of the ideal array shape under steady tow conditions. In such cases adaptive algorithms outperform the conventional Kalman filter-based ones.

Lainiotis et al.³⁶, implemented an algorithm, based on the generic Multi-Model Partitioning filtering theory of Lainiotis, in the problem of towed array shape estimation is proposed with satisfactory results.

In general, the generic MMP approach has been employed successfully and, practically important and performance efficient classes of passive underwater target tracking algorithms have been developed. These classes contain non-adaptive as well as adaptive target tracking algorithms, for both maneuvering and non-maneuvering targets. Both classes of algorithms are highly beneficial over previously reported ones with regards to performance. The exploitation of the parallelism inherent in these algorithms leads to implementations highly efficient from a computational point of view.

3.1.3 Real-Time Ship Motion Estimation

Accurate and real-time estimation of a ship's motion is of great importance to many ship-related problems, such as, ship steering, dynamic ship positioning, aircraft takeoff/landing in carriers, marine oil exploration, and offshore platforms. In these cases it is desirable to have accurate estimates and predictions of the motions, velocities, and accelerations of the ship, based on knowledge of the ship's dynamics and on noisy measurements of speed, position, sea state, and other quantities³⁷.

Unfortunately, ship dynamics are in reality time varying and dependent upon exogenous factors, such as for example the sea state. Therefore, most approaches, in order to use the Kalman filter theory, are based on simplifying assumptions, which sometimes render them completely inappropriate for any practical purpose. Kalman filter theory is known to produce low quality estimates when the mathematical model used to describe the physical problem, is in effect in disagreement with reality. Moreover, a fact usually overlooked (simply because the Kalman filter theory cannot cope with it) is that the initial conditions of any filter designed to operate on a ship model may not be initialized with Gaussian initial conditions.

The use of the Multi-Model Partitioning theory in designing filters for ship motion estimation, under time varying sea conditions and non-Gaussian initial conditions, is demonstrated by Lainiotis et al.³⁷⁻³⁹. Their work focused on those components of a ship's motion that are of concern to a filter designer such as, the heave and the roll, i.e., the translational motion along the vertical axis of the ship and the rotational motion along the longitudinal axis of the ship, respectively. This leads to a deconvolution problem, where several techniques have been proposed. One of the most advertised and modern deconvolution techniques is Minimum Variance Deconvolution, whereby results of state-space estimation and Kalman filter theory are brought to bear. When faced with the deconvolution problem the researchers developed, based on the generic Multi-Model Partitioning approach of Lainiotis, two classes of algorithms for solving deconvolution problems: a non-adaptive class and an adaptive class. The former class is used when all system parameters and the system input are completely known, whereas the latter class is used when our knowledge of the system's characteristics and/or of the system input is incomplete.

Their results show that the non-adaptive class is preferable over previously reported techniques in multisensor cases. The adaptive class of algorithms is naturally more computationally involved, but by using parallel processing capabilities currently available and by exploiting the parallelism inherent in these algorithms, the computational overhead is dramatically increased. In any case, the overall performance/computational complexity ratio is favorable. Both classes of algorithms are capable of effectively coping with non-Gaussian initial state statistics and highly beneficial over previously reported ones with regards to performance.

Further work of Lainiotis et al.⁴⁰⁻⁴¹ investigated the combined application of the Multi-Model Partitioning theory and Neural Networks for heave compensation in ship position estimation.

3.1.4 Direction of Arrival (DOA) Estimation

The problem of direction of arrival (DOA) estimation given a set of measurements of the output of a sensor array has been a topic of considerable interest in the literature. Much of the recent work in array processing has focused in developing high resolution methods for determining the incident angles of plane waves received by an array of sensors. This problem has important applications in areas such as radar, sonar, radio and microwave communication, underwater acoustics and source localization, and geophysics. The schematic representation of the Direction of Arrival (DOA) problem is shown in figure 5.

By considering the above m element array of sensors and n far-field point sources, we can define the $(m \times 1)$ vector $a(\varphi_i)$ to be the complex array response $\alpha(\varphi_i) = [1, e^{-j\omega_i \tau_i}, e^{-j\omega_i 2\tau_i}, \dots, e^{-j\omega_i (m-1)\tau_i}]^T$ for a source at direction φ_i . Assuming that n signals are simultaneously intercepted the m sensors, under the narrowband assumption, the array output $z(t)$ is modeled by an equation of the form:

$$z(t) = \begin{bmatrix} \sum_{i=1}^n e^{j2\pi(c \cdot t / \lambda_i)} \\ \sum_{i=1}^n e^{j(2\pi(c \cdot t / \lambda_i) - 1 \cdot 2\pi(d \sin(\varphi_i) / \lambda_i))} \\ \vdots \\ \sum_{i=1}^n e^{j(2\pi(c \cdot t / \lambda_i) - (m-1) \cdot 2\pi(d \sin(\varphi_i) / \lambda_i))} \end{bmatrix} + \mathbf{v}(t) \quad (20)$$

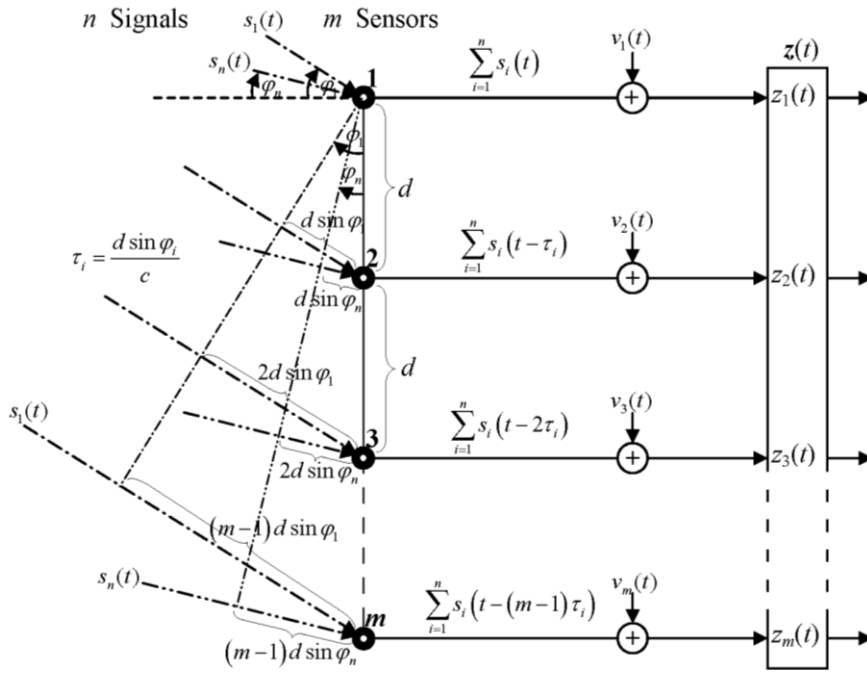


Figure 5. Schematic representation of the Direction of Arrival (DOA) problem.

Most of the proposed solutions employ the Maximum Likelihood (ML) approach. Sub-optimal techniques with reduced computational load are also quite popular, such as the Minimum Variance (MV) method, the MUSIC

method, the related Minimum Norm method, the ESPRIT estimator and the weighted subspace fitting (WSF) algorithm. These methods are either off-line or two-step processing methods and their performance is critically dependent on the validity of their underlying assumptions.

Moreover, the basic assumption made when such methods are employed is that the number of sources is known. In many practical situations, however, this prior knowledge may well be unavailable. One possible approach to the solution of this detection problem is based on the application of information theoretic criteria for model order selection such as, the Final Prediction Error (FPE) criterion, the Akaike's Information Criterion (AIC) and the Minimum Description Length (MDL) Criterion. Most of the techniques that result from the above criteria are based on and restricted by the assumption that the data are Gaussian and upon asymptotic results. In any case, situations when the number of emitting sources is varying in time cannot efficiently be handled by existing DOA methods.

The proposed approach^{47,48,49}, addresses the combined problem of detecting the number of emitting sources and estimating their directions of arrival and as a side effect, also estimates the received signal envelopes. The problem is first transformed into the time-domain, allowing all the powerful arsenal of related filtering techniques to be brought to bear. The problem is thus reformulated, so that the measurement equation is expressed as a non-linear function of the extended location vector, which is augmented to contain the source locations as well as the emitted signals.

Two well known advanced estimation techniques are combined, the Multi Model Partitioning (MMP) approach^{2,8} with the Extended Kalman Filter (EKF)², for general (not necessarily Gaussian) data PDFs. A bank of EKFs is implemented, each matching a different conditional model.

The Multi Model Partitioning Filter is used to evaluate the conditional models, by computing the a posteriori probability that each candidate model is the correct one. The decision is based on the maximum a posteriori probability (MAP) criterion. The overall estimate of the MMPF can be taken either to be the individual estimate of the elemental filter exhibiting the highest posterior probability (called "MAP –Maximum A Posteriori-estimate") or the weighted average of the estimates of the elemental filters, where the weights are simply the posterior probabilities associated with each estimate (called "MMSE – Minimum Mean Square- estimate")^{28,13}.

The proposed method addressed the general problem of the DOA estimation from a new perspective by simultaneously estimating the number of sources, as well as the directions of arrival and the signals emitted. The reformulation of the problem led to a nonlinear state-space model with partially unknown structure and a number of different models that possibly

fit the data and the multiple models are evaluated, using the Multi-Model Partitioning algorithm. The method is adaptive, as it is not only able to identify the correct number of sources, but to track changes in the model structure in real time as well. Thus, the method handles also successfully the problem of a variable number of sources (figure 6). Finally, note that the algorithm exhibits a high degree of parallelism; thus, it can be implemented in a parallel processing environment.

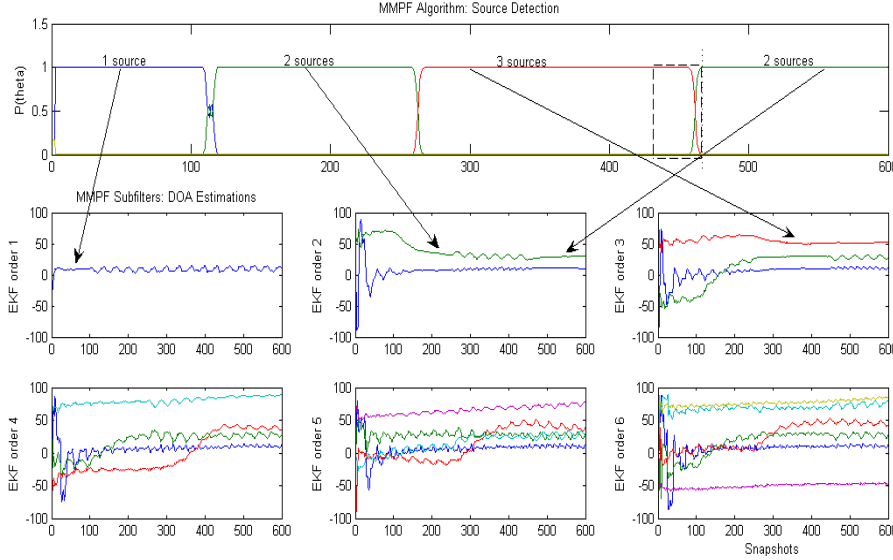


Figure 6. Detection and estimation results for 1-3 sources at directions 10°, 30° and 50°. Dotted lines indicate a sliding window of 40 snapshots used with conventional algorithms.

3.2 Oil Exploration and Seismic Signal Processing

One of the popular methods used nowadays for oil exploration is the seismic method. Basically it consists of artificially generating seismic waves (wavelets), measuring the reflected signal, and processing it to determine the nature of the subsurface layers. This processing is performed using techniques cumulatively known as deconvolution techniques, the name stemming from the fact that they reproduce a system's input based on knowledge of the system's transfer function characteristics and of the system's response.

Although several deconvolution techniques have been proposed at times (e.g. Minimum Variance Deconvolution), they do not compensate model uncertainty that can be either, parametric due to an unknown finite-dimensional parameter vector, or, structural due to an unknown functional form of the model⁴².

The signal received by a seismic sensor (often called the seismic trace) is described by the following convolution summation model,

$$z(k) = V_R(k) + n(k) = \sum_{j=1}^k \mu(j)V^+(k-j) + n(k) \quad (21)$$

where $V_R(k)$ is the noise-free seismic trace, $n(k)$ is measurement noise, $V^+(i)$ is a sequence associated with the basic seismic wavelet and $\mu(j)$ is the reflection coefficient sequence, assumed random, zero-mean and white. The signal $V_R(k)$ is a superposition of wavelet replicas reflected from the interfaces of earth's subsurface layers, while the $\mu(j)$'s are related to interface reflection and transmission coefficients. Our objective is to determine the values of the $\mu(j)$ sequence. The above model is shown⁴³ to be equivalent to a state-space set of model equations presented in section 2.1.

The possibility that the filter designer may be faced with the task of designing an optimal filter in the face of incomplete model knowledge occurs more often than not in practice, since it is virtually impossible to accurately model any physical process, especially when this process is inherently nonlinear and/or nonstationary, as is the case with geophysical exploration methods.

Katsikas and Lainiotis in⁴² developed two classes of algorithms for solving the seismic signal deconvolution problem: a non-adaptive class and an adaptive class, as well as a distributed version of the non-adaptive class. All algorithms have been developed by using the generic Multi-Model Partitioning approach^{2,4}.

The non-adaptive class is used when all system parameters and the system input are completely known, whereas the adaptive class is used when our knowledge of the system's characteristics and/or of the system input is incomplete. The algorithms derived with the use of the Lainiotis filter and smoother have been found to be superior in performance, especially in high SNRs, and superior in computational efficiency in multisensor cases that are exactly the situations of practical interest.

A distributed version of the algorithm is also developed. In geophysical applications the number of sensors (geophones) varies from a few hundred to several thousands and the sensors are usually located in different local subsystems (geophone clusters). A distributed algorithm allows local measurements to be processed near the sensing devices and the generated local estimates are then communicated to the central facility for further processing (figure 7).

The adaptive class of algorithms is naturally more computationally involved, but by using parallel processing capabilities currently available and by exploiting the parallelism inherent in these algorithms, the computational overhead is dramatically increased in terms of execution speed. In any case, the overall performance/computational complexity ratio is favorable. The distributed version of the non-adaptive algorithm can be four orders of magnitude faster than its centralized counterpart.

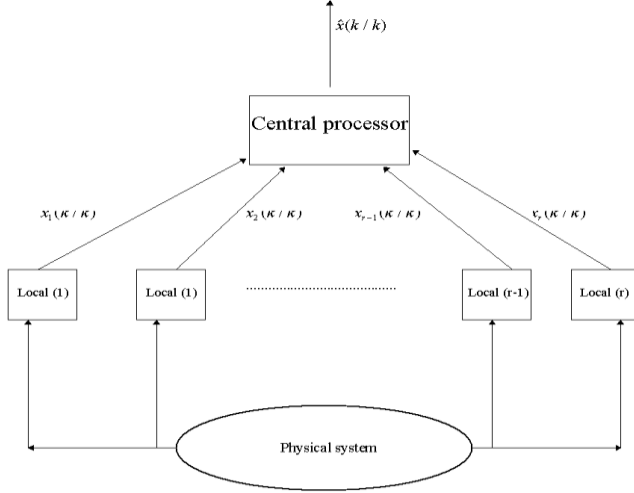


Figure 7. Distributed processing architecture of the proposed Multi-Model Partitioning algorithms.

3.3 Structural Reliability and NDE

Fatigue crack analysis is an essential tool for life prediction and maintenance of structural components. Lifetime predictions and in-service inspections of each component are used to update the reliability analysis of the overall structure. Fatigue crack growth (FCG) monitoring and failure prediction are critical in numerous engineering applications especially in any rare, expensive, or, dangerous structure that is impossible to test a priori in statistically large samples.

The typical semi-empirical models currently used to describe FCG are nonlinear functions of the crack size a , the number of fatigue cycles N , the stress intensity factor ΔK , and several parameters of the material, the component geometry or external conditions (n, C, Y, \dots) ⁴⁴ e.g.:

$$da/dN = C[\Delta K(a)]^n = C[\Delta S(\pi a)^{1/2} Y]^n = g(a, C, n, \Delta S, Y, \dots) \quad (21)$$

$$N_{k+1} - N_k = \Delta a / g(a_k, C, n, \dots) \Rightarrow N_{k+1} = N_k + f_R(a_k, \Delta a_k, C, n, \dots) \quad (22)$$

For practical applications it is vitally important to have on-line, real-time monitoring and on-line estimation/identification of the FCG, in order to obtain earlier and more accurate predictions of remaining lifetime to failure. The above FCG equations are written in an integrated form that calculates the number of fatigue cycles required for a specific increase of the defect Δa :

The crack state evolution is usually observed by a Non-Destructive Evaluation (NDE) method (microscope, X-rays, ultrasonics, acoustic or thermal emissions, etc.) that reports the crack size at every inspection. The observation equation of the NDE method is in general nonlinear and with a lot of uncertainties. In order to develop an advanced method for FCG identification and prediction, Moussas et al.⁴⁵ combined the Crack Growth and the Non-Destructive equations in the following nonlinear/parametric state-space model form, that is suitable for use by the Multi-Model Partitioning techniques.

$$x_{k+1} = f(x_k) + w_k : \begin{bmatrix} N \\ a \\ \Delta a \end{bmatrix}_{k+1} = \begin{bmatrix} N + f(a, \Delta a) \\ a + \Delta a \\ \Delta a \end{bmatrix}_k + \begin{bmatrix} w_N \\ w_a \\ w_{\Delta a} \end{bmatrix}_k \quad (23)$$

$$z_k = h[k, x_k] + v_k \quad (24)$$

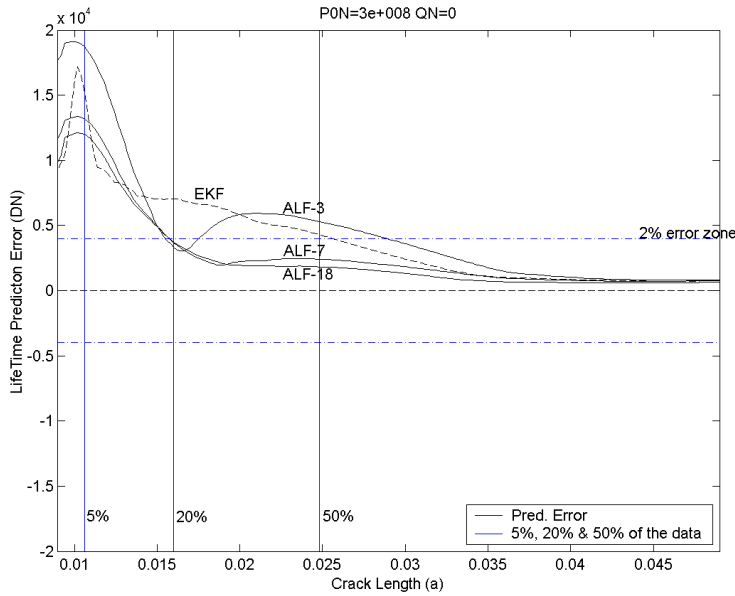


Figure 8. EKF & MMPA average prediction errors over 10 experiments. Three MMPA implementations are shown with 3, 7 and 18 filters respectively (MMPA satisfies the 2% error limit earlier using less than 20% of the data, EKF crosses the same limit much later using more than 50% of the data).

Several implementation of the Multi-Model Partitioning algorithm have been developed for FCG model identification, crack growth prediction and subsequently, residual lifetime estimation.

The MMP algorithms are tested against the adaptive EKF with augmented state. Both filters use the non-linear state-space representation of the same FCG law, and they process real experimental FCG data (a vs. N).

The results show that both predictors (MMPA & EKF) do converge to the actual time to live. However, the MMPA can do it more accurately and much sooner than the EKF, thus requiring fewer measurements and leaving more time for reaction before a catastrophic failure occurs (figure 8).

Due to its partitioned structure, the MMPA overcomes easily its complexity as it is also suitable for parallel implementation. In addition, the MMPA is more robust than any single EKF, as it incorporates the mechanism to isolate any diverging sub-filter from its filter bank.

3.4 Environmental Monitoring

The remote sensing of atmospheric and oceanic properties in both active and passive models has been traditionally limited due to the nature of the classical instrumentation available. The insufficient penetration of infrared and microwave radiation, especially through the water, has restricted most of the oceanic studies to surface characteristics. Among alternate observation procedures currently available, the most viable method is that of obtaining vertical profiles of radar-like range gated systems utilizing lasers as the radiation emitting source. Such laser systems are referred to as LIDAR (Laser Integrated Radar).

3.4.1 LIDAR Signal Processing

LIDAR systems constitute an engineering problem of great practical importance in environmental monitoring sciences. They are usually installed and operate from aircraft or space, and they provide indispensable data to both oceanographic and climatic studies. LIDAR systems operate near appropriate wavelengths and have sufficient penetration of the order of tens of meters under favorable conditions.

Typically, atmospheric parameter estimation has entailed signal processing techniques that are based on single-pulse LIDAR returns. The

state-variable formulation of the problem and the related linear Kalman filter and the nonlinear extended Kalman filter (EKF) have been applied to the estimation of the return power and to the logarithm of the return power for incoherent backscatter LIDAR, in which multiplicative noise or speckle is present. Several drawbacks are usually associated with LIDAR state-variable models. The whiteness assumption of the Kalman theory is violated, as the multiplicative noise source, seen in the speckle, exhibits serial correlation, and, in addition, the same quantity demonstrates non-Gaussian statistics. In addition, the environment around the measurement may abruptly introduce unknown bias effects in the observation sequence, or, from the system's reference point, random failures may suddenly occur. Finally, system parameters or part of the signal structure is usually unknown, which requires adaptive filter designs for their determination.

Clearly, signal processing for LIDAR applications involves highly nonlinear models and consequently nonlinear filtering; however, optimal nonlinear filters are practically unrealizable. The nonlinear state-space representation of a possible LIDAR system model, in the presence of both additive and multiplicative noise (speckle) and with particular reference to estimation of the log-power returns, is presented by the following two-dimensional form:

$$x_1(k+1) = x_1(k) + \Theta_w w_1(k) \quad (25)$$

$$x_2(k+1) = 1 + w_2(k) \quad (26)$$

$$z(k+1) = \Theta_z x_2(k+1) \exp[x_1(k+1)] + v_1(k+1) \quad (27)$$

where, the first state, $x_1(k)$, is the power return and the second, $x_2(k)$, is the speckle; $w_1(k)$ is a white Gaussian sequence with zero mean and unity variance that is scaled to $Q_1(k)$ by the unknown parameter Θ_w ; $w_2(k)$ is a zero mean white Gaussian sequence independent from $w_1(k)$, having covariance $Q_2(k)$; the state return is assumed to arrive corrupted by additive Gaussian noise, $v(k)$, with zero mean and covariance $R(k)$; Θ_z represents an unknown parameter to be identified. In meteorological measurements, the strength of the additive stochastic disturbance associated with equation (25) is unknown, and hence the purpose of the quantity Θ_w becomes meaningful and effective. Expressions (25)-(27) are in a form which permits discrete nonlinear state estimation and identification.

Based on the above model structure, the Lainiotis' Multi-Model Partitioning methodology and the related nonlinear and adaptive filtering algorithms have been effectively applied to LIDAR signal processing^{9,46}. The authors defined a vector: $\Theta = [\Theta_z \quad \Theta_w]$, that contains all model uncertainties. By partitioning the parameter space of vector Θ , they designed

a Multi-Model Partitioning algorithm with nonlinear sub-filters that matched the different Θ realizations. A state-augmented EKF identifier was also designed by including the parameter vector Θ in its state vector.

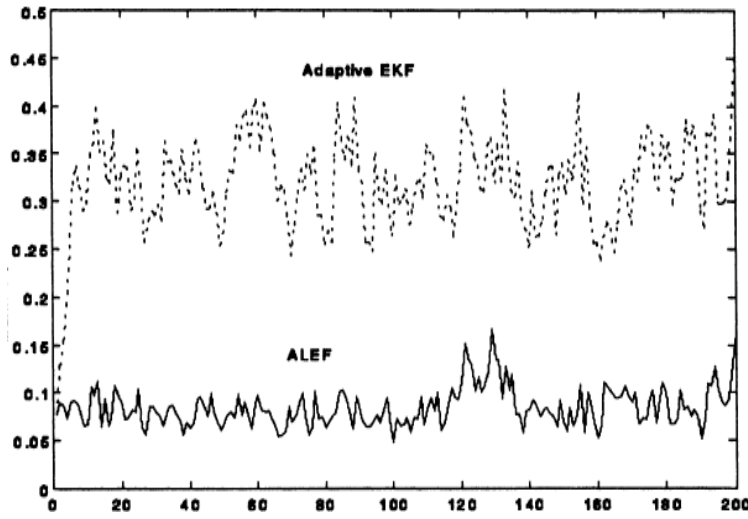


Figure 9. Log-Power estimation with multiplicative noise (Mean Square Error vs Time Steps) of the Adaptive (augmented) EKF and the Adaptive Lainiotis MMP Filter (ALEF), when observation matrix is unknown⁹.

The resulted Multi-Model Partitioning algorithm found to be very effective and significantly superior to the nonlinear extended Kalman filter (EKF), which has been the standard nonlinear filter in similar engineering applications. Especially when the model is not completely known, while the mismatched EKFs develop significant bias errors, the adaptive MMP Lainiotis estimator adjusts within a few time steps and eliminates the bias error (figure 9). Moreover, it outperforms the state-augmented EKF as the latter exhibits a slow adaptation response.

3.5 Time Series Order Identification

Time Series (ARMA) models have found great attention in speech analysis, biomedical applications, hydrology, electric power systems, financial time series prediction and many other areas such as multi-channel data analysis, epilepsy research, EEG and ECG analysis, geophysical data processing, clutter suppression in airborne radar signal processing, etc.

A typical multi-variate (MV) AR model of order θ [AR(θ) model] for a stationary time series of vectors observed at equally spaced n instants is defined as:

$$y_n = \sum_{i=1}^{\theta} A_i y_{n-1} + v_n, \quad E[v_n v_n^T] = R \quad (28)$$

where the m -dimensional vector v_n is uncorrelated random noise vector with zero mean and covariance matrix R , and A_1, \dots, A_{θ} are the $(m \times m)$ coefficient matrices of the AR model.

The problem can be described as follows: given a set of samples from a discrete time process $\{y(k), 0 \leq k \leq N-1\}$, it is desired to obtain the set of coefficients $\{A_i\}$ which yields the best linear prediction of $y(N)$ based on all past samples:

$$\hat{y}(N/N-1) = \sum_{i=1}^{\theta} A_i y(N-1) \quad (29)$$

where $(N/N-1)$ denotes the predicted value of $y(N)$ based on the measurements up to and including $y(N-1)$.

It is obvious that the problem is twofold. The first and most important task of this problem is the successful determination of the predictor's order θ . Once this task is completed, one proceeds with the computation of the predictor's matrix coefficients A_i .

Determining the order of the process is usually the most delicate and crucial part of the problem. Over the past years substantial literature has been produced for this problem and various different criteria, such as Akaike's, Rissanen's, Schwarz's, Wax's have been proposed to implement the order selection process⁵⁰. The above mentioned criteria are not optimal and are also known to suffer from deficiencies; for example, Akaike's information criterion suffers from overfit. Also their performance depends on the assumption that the data are Gaussian and upon asymptotic results. In addition to this, their applicability is justified only for large samples; furthermore, they are two pass methods, so they cannot be used in an on line or adaptive fashion.

The Multi-Model Partitioning method for simultaneous order identification and parameter estimation proposed by Katsikas⁵¹, Likiothanassis and Lainiotis (1990) managed to overcome the above deficiencies. They first considered the problem of simultaneous selection of the AR model order and of the AR parameters identification for the scalar case. The method is based on the well known adaptive Multi-Model Partitioning theory, it is not restricted to the Gaussian case, it is applicable to on line/adaptive operation and it is computationally efficient. Furthermore, it identifies the correct model order very fast.

The proposed Multi-Model Partitioning method is also extended for simultaneous order identification and parameter estimation of multivariate (MV) autoregressive (AR) models⁵². The proposed methods succeed to select the correct model order and estimate the parameters accurately in very few steps and even with a small sample size.

Compared to many other well established order selection criteria (figure 10) , namely Akaike's Information Criterion (AIC), Schwarz's Bayesian Information Criterion (BIC), Hannan's and Quinn's (H&Q), Brockwell's and Davis' and C. C. Chen, R. A. Davis and P. J. Brockwell order determination criteria (DC, MDC), the proposed MMP method requires a fraction of the data in order to produce the same or better results⁵².

ORDER ESTIMATION WITH SAMPLE SIZES 50, 100 (IN PARENTHESIS) AND 200 (SQUARE BRACKETS)			
Criterion	Estimated Order		
	0-2	3	4 – 10
<i>AIC</i>	10 (0) [0]	62 (88) [92]	28(12)[8]
<i>AICC</i>	38 (2) [0]	62 (96) [96]	0 (2) [4]
<i>H&Q</i>	32 (2) [0]	62 (98) [99]	6 (0) [1]
<i>BIC</i>	68 (16) [0]	32 (84) [100]	0 (0) [0]
<i>DC</i>	22 (4) [0]	66 (88) [99]	12 (8) [1]
<i>MDC</i>	26 (2) [0]	63 (92) [100]	11 (6) [0]
<i>MMPF</i>	0 (0) [0]	100 (100) [100]	0 (0) [0]

Figure 10. The above table⁵² summarizes the comparison results and shows that: classical order selection criteria, as the model order increases, require a larger data set in order to achieve a satisfactory performance, while MMPF is consistent and 100% successful for all data set sizes.

In addition, the MMPF algorithm is also successful in tracking model order changes in real time (figure 11) [52].

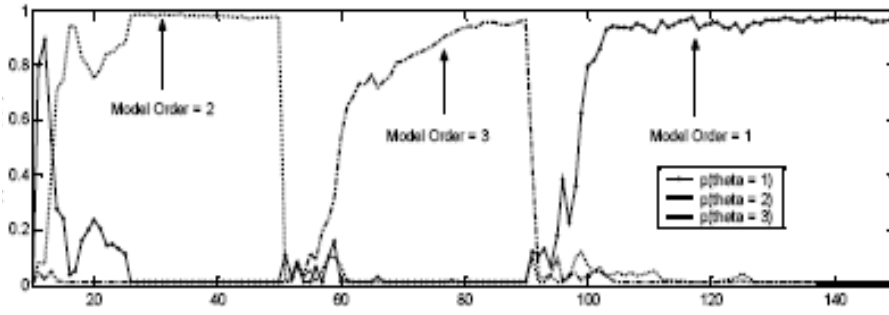


Figure 11. MMPF tracking model order changes in real time ($p(\theta/k)$ vs. k) [52].

A more general approach to the problem is attempted⁵³ by applying the Evolutionary Multi-Model Partitioning methods discussed in the previous Section. The authors combined the Multi-Model Partitioning approach with the Genetic Algorithms thus overcoming the difficulties with the initial set of candidate filters. Even when the optimal value is not present in the initial parameter set, it is eventually produced by the GA that generates new possible solutions. The Evolutionary MMP Algorithms applied for order identification of ARMA and AR discrete time systems, performed significantly better than the conventional MMP approach (figure 12).

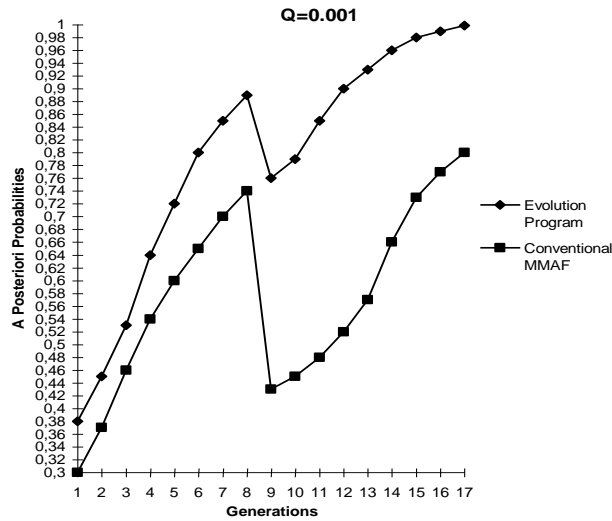


Figure 12. Evolution of the a-posteriori probabilities of the best genome (Evolutionary approach) compared to the conventional Multi-Model approach (MMAF), with model order change during operation⁵³.

3.5.1 Traffic Pattern and Anomaly Detection

With the rapid expansion of computer networks, security has become a crucial issue. Intrusion Detection Systems (IDS) are being designed to protect such critical networked systems. There are two major approaches in intrusion detection: anomaly detection and misuse detection. The main advantage with anomaly detection is that it can detect new forms of attacks or network misuse, as they will probably deviate from the normal behavior.

The proposed method⁵⁴ uses simple and widely found datasets, i.e. from bandwidth utilization, in order to learn normal network utilization patterns. Once a set of known network working conditions is prepared, an adaptive Multi-Model Partitioning algorithm is applied to identify the current working conditions detect any unusual events.

The proposed method has two advantages, first, it is based on a powerful multi model partitioning algorithm, (MMPA) proposed by Lainiotis, known for its stability and well established in identification and modeling, and secondly, its observations come from utilization datasets that are easy to find and collect. The method initially prepares some Time-Series or State-Space models, based on past datasets that will represent various traffic patterns of a network connection.

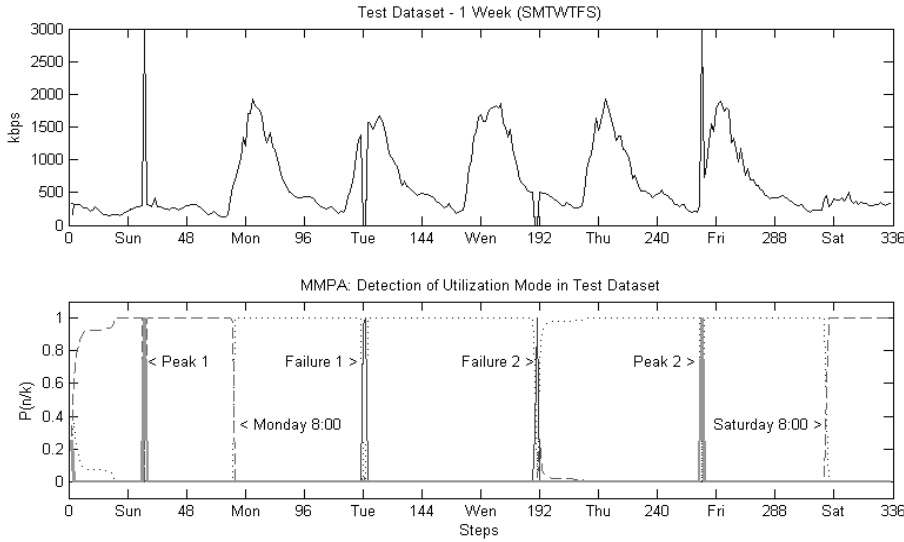


Figure 13. Test dataset for 1 week (SMTWTFS) containing peaks and failures (up), and, the MMPA successful detection of the changes and anomalies in the dataset (down).

Subsequently, the adaptive Multi-Model Partitioning algorithm is applied to processes the new traffic data received from the network, in order to detect deviations (anomalies) from the typical conditions.

As shown using real traffic⁵⁴, the MMP algorithm detects correctly all changes, failures or unusual activities included in the test datasets. The method is also very fast and it can perform equally well off-line and in real-time.

3.6 Neural Network Structure Identification

A continuing question in the research of neural networks is the size of a neural network required to solve a specific problem. If the training starts with a small network it is possible that no learning is achieved. On the other hand, if a larger than required network is used, then the learning process can be very slow and/or overfitting may occur. Furthermore, the underlying model, that provides the data for the training set, is usually unknown or variable, resulting to incomplete driving information. In these cases, no standard rules exist, on how one can implement a network which will solve a specific problem⁵⁵.

In the literature, there are several methods reporting how one can face this problem, by minimizing the size of the network and yet maintaining good performance. One may achieve these design objectives in one of two ways: (a) Network growing, where we start with a small network and we add a new neuron or a new layer of hidden neurons only when we are unable to meet the design specification. (b) Network pruning, in which case we start with a large network with an adequate performance for the problem at hand and then prune it by weakening or eliminating certain synaptic weights or neurons, in a selective and orderly fashion.

The more significant disadvantage of the above mentioned methods is that they demand the a-priori knowledge of the data record (training set), thus they are off-line techniques. In real time applications, such as time series prediction, the whole measurement set may not be available from the beginning. Furthermore, in this case one must know the specific number of inputs required to predict the output. In the general case there is not a standard procedure to determine the number of the previous inputs (i.e. the order of the time series or the number of the network's inputs), that is necessary to perform "perfect" prediction. Furthermore, these methods can not adapt the network size if the underlying model that supports the data, change order during the operation so they work under the Gaussian assumption.

In an attempt to face the above problems an adaptive approach is proposed based on the Lainiotis Multi-Model Partitioning theory. The basic idea originates from the work performed in⁵¹ for the problem of Auto Regressive model identification with unknown process order, discussed in the previous section.

The method faces the problem as a global non-linear identification problem, the solution of which requires the optimization of a certain criterion. The solution to this problem is given by a combination of the multi-model partitioning theory with a Localized EKF (LEKF). Specifically a bank of Kalman filters is realized, each fitting a different order model. Each of these filters is implemented as a specific neuron using as training algorithm the Localized implementations of the EKF. In sequence, an ALF algorithm is realized using a type of neuron, called ALF neuron. It is placed in the output layer of a NN and it is connected to M LEKF neurons in the hidden layer, representing the bank of the M ‘candidate’ models.

The advantage of the proposed method, which integrates in a self-organized neural network the localized EKF algorithm with the framework of multi-model partitioning theory, is that identifies the structure of a dynamical system in one-pass, on-line (adaptive) fashion. The resulting neural networks are recurrent and adaptive, in the sense that they have the ability of tracking successfully the changes in the model structure, in real time^{56,57}.

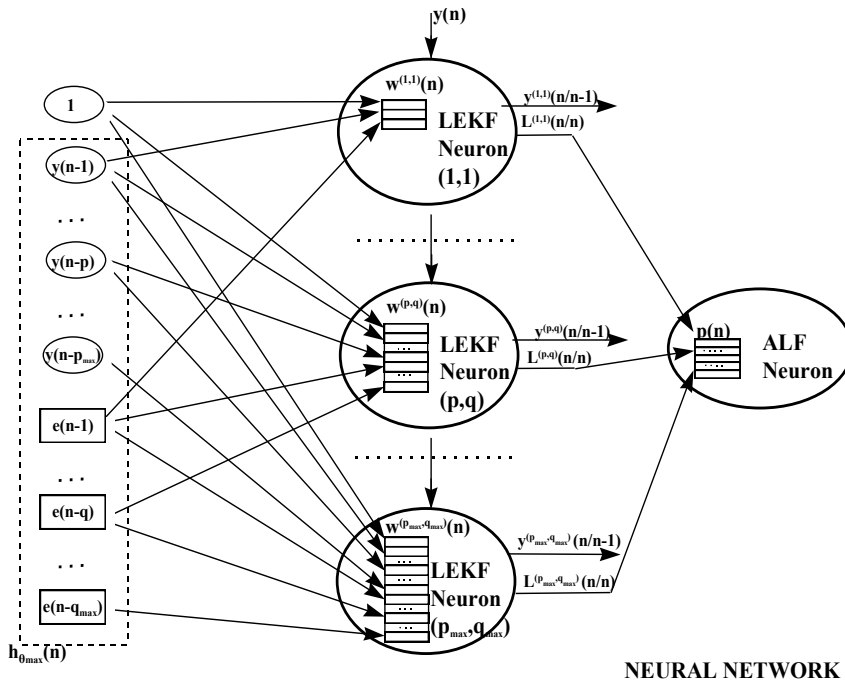


Figure 14. Structure of the a Self-organized NN for structure identification.

In addition, the MMP method has the powerful characteristic that all filters required by ALF interact independently, which makes the method amenable for parallel and VLSI implementations.

A further improvement of this method is presented in⁵⁸ where a more general method for optimizing the structure and the weights of a fully connected neural network, is developed. Typical Multi-Model Partitioning methods depend on the a priori selection of the set of conditional models, and they give near optimal solutions when the true order of the model does not belong to the initial population of the candidate models. This disadvantage is alleviated using natural selection techniques, such as the Genetic Algorithms (GAs), which are among the best known methods for searching and optimization. As also discussed in Section 2, this new evolutionary method combines the effectiveness of the MMP theory with the robustness of the GAs.

4. SUMMARY

A concise review of the theory underlying the Multi-Model Partitioning approach is presented. The Multi-Model Partitioning theory was introduced by Lainiotis forty years ago. It has received a great deal of attention due to its success in decomposing complex problems into simpler sub-problems, and in handling effectively structural or parametric uncertainties. Since then, three generations of multi-model partitioning algorithms have appeared and numerous applications of the multi-model partitioning approach have been developed. A brief survey of selected applications of the approach is also presented.

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