# DESIGN OF A MULTIPLE MODEL SIMULATION TEST-BED FOR A COMMON-BASED COMPARISON OF TRACKING FILTERS

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**Abstract.** During the last three decades, the application of state-space estimation methods to the target tracking and trajectory estimation problems created a lot of new tracking filters or modified old filters. Unfortunately all these filters were tested separately in many different environments. This renders difficult and often impossible the direct comparison of the filters, due to the incompatibility of the available results. A solution can be found by creating a unified environment, for the testing and the comparing of the various tracking filters. From this point of view, the paper presents: first, a collection of the more often used state-space models and test cases for the target tracking problem, second, a selection of the most representative of them, in order to create a general target tracking test-bed (TTTB), as a common base for the test and the comparison of different tracking filters, and finally, a sample set of tracking filters applied and tested using the proposed unified framework.

### 1. INTRODUCTION

In order to use advanced methods for trajectory estimation and target identification, a mathematical model for the description of the target-radar system, and a filtering algorithm for the state estimation, are needed. During the last decades, various state-space models and filtering methods have been proposed by the researchers. These models and filters vary in design, being simple and linear to more sophisticated and non-linear. Some of these tracking fillers have been designed for special applications, and others for more general use. Each candidate filter is tested either in real or simulated situations for performance evaluation.

The choice of a tracking filter for a particular application depends mainly on: the target dynamics, the radar/sensor dynamics, the accuracy requirements and the available computational resources. The main aim in each situation is to use the simplest suitable model and filter which will achieve the best results. This means that all filters must be tested under the same conditions (environment), and compared with each other. This is a very important point. The selection of the testing environment affects the compatibility of present and previous results. Arbitrary selection of testing environments would lead to the repetition of work by another researcher, under other, perhaps arbitrary, testing conditions. In fact, it is almost impossible to make a direct comparison between already available results.

A solution which leads toward the compatibility of tests for tracking filters is presented in this paper. First, a collection of the different models, parameter values, trajectory schemes, test cases, used during the last decades is made. Then, a comparison and a selection follows, in order to create a satisfactory test-bed for the target tracking filters (TTTB).

### **2** FORMULATION OF THE PROBLEM

The problem can be divided in two main parts. One is the system's model, or in other words, the mathematical representation of the target-radar system. The other is the filtering algorithm, which contains the mathematical formulae for the processing of the data.

### 2.1 The Target-Radar System Model

As the target tracking problem is a state estimation problem, its model can be represented by the following equations:

$$X(k+1) = f(X(k),k) + g(X(k),k)W(k)$$
(2.1)

$$Z(k) = h(X(k), k) + V(k)$$
(2.2)

where, W(k) and V(k) are the input and the measurement noise processes respectively.

One is interested in estimating the target's state X(k) based on all measurements Z(l), l = 1, 2, ..., k. Equation (2.1) represents the target dynamics. The state vector X(k) contains the target position (x, y, z, r, b, ...), velocity, and acceleration. Equation (2.2) is the radar's measurement equation and the measurement vector Z(k) contains distance and/or angle measurements.

The noise processes W(k) and V(k) are assumed to be zero-mean white Gaussian noise processes. The statistics Q(X(k),k) of W(k) are selected to compensate modelling errors the statistics R(X(k),k) of V(k) should be selected to represent all possible deviations such as measurement biases, false measurements, noise, etc. The system dynamics can be described in a Cartesian and/or spherical coordinate system (fig. 1). As we will see, the linearity of the model depends also on this choice.

#### 2.2 The Tracking Algorithms

The tracking filter computes a smoothened estimation of the target's present position and velocity, as well as a prediction of the next scan. A simple method of computing these quantities is the so-called a- $\beta$  tracker [ref.], which calculates the present smoothened target position and velocity by:

$$x(k) = x_{p}(k) + \alpha [x_{m}(k) - x_{p}(k)]$$
(2.3)

$$\dot{x}(k) = \dot{x}(k-1) + \frac{\beta}{T} [x_m(k) - x_p(k)]$$
(2.4)

and the predicted position at the *k*+1st scan by:

$$x_{p}(k+1) = x(k) + \dot{x}(k)T$$
(2.5)

where,  $x_p(k)$  is the predicted position of the target at the kth scan,  $x_m(k)$  is the measured position,  $\alpha$  is the position smoothing parameter,  $\beta$  is the velocity smoothing parameter, and *T* is the radar's sampling interval. If acceleration is needed, a third equation can be added to calculate this. The filter is then called the  $\alpha$ - $\beta$ - $\gamma$  tracker.

The above tracking filter needs only the values of  $\alpha$  and  $\beta$ . If  $\alpha = \beta = 0$ , the tracker uses no current information. If  $\alpha = \beta = 1$ , no smoothing is included. In the first case one may have large bias errors, and in the second large random errors. This means that the values for  $\alpha$  and  $\beta$  must be somewhere between 1 and 0.

As the means for choosing  $\alpha$  and  $\beta$  become more sophisticated, the optimal  $\alpha$ - $\beta$  tracker becomes equivalent to the Kalman filter. This filter can handle a dynamic or manoeuvring target due to its inherent ability to take into consideration of manoeuvre statistics. Knowledge of the statistics of measurement noise and target dynamics, is provided by the model of the target-radar system (eq. (2.1), (2.2)).

When the model is non-linear then more sophisticated filters are needed as the extended Kalman filter (EKF) or adaptive and multi-model filters. These filters are able to deal with non-linearity and parameter uncertainty.



Figure 1. Cartesian (a) and Spherical (b) coordinate system

## 3. TARGET DYNAMICS

Equation (2.1) represents the target dynamics. For some targets, a constant velocity model is sufficient, but for others, an acceleration term is needed. The target dynamics can be described in a two-dimensional or three-dimensional Cartesian or spherical coordinate system. The state vector X(k) can take different forms such as:

$$X(k): \begin{bmatrix} x \\ \dot{x} \\ \dot{y} \\ \dot{y} \end{bmatrix}, \begin{bmatrix} x \\ \dot{x} \\ \dot{x} \\ \dot{y} \\ \dot{y} \end{bmatrix}, \begin{bmatrix} x \\ \dot{x} \\ \dot{x} \\ \dot{y} \\ \dot{y} \\ \dot{y} \end{bmatrix}, \begin{bmatrix} x \\ \dot{x} \\ \dot{x} \\ \dot{y} \\ \dot{y} \\ \dot{y} \\ \dot{y} \\ \dot{z} \\ \dot{z} \end{bmatrix}, \begin{bmatrix} r \\ \dot{x} \\ \dot{x} \\ \dot{y} \\ \dot{y} \\ \dot{y} \\ \dot{y} \\ \dot{z} \\ \dot{z} \end{bmatrix}, \begin{bmatrix} r \\ \dot{r} \\ \dot{r} \\ \dot{r} \\ \dot{b} \\ \dot{c} \\ \dot{$$

where, r, b, e stand for Range, Bearing (Azimuth), and Elevation.

### 3.1 Linear Equations for Target Dynamics

When the target's behaviour is described by a linear model, the equation (2.1) has he following form :

$$X(k+1) = F(k)X(k) + G(k)W(k)$$
(3.2)

where,  $W(k) : N\{0, Q(k)\}$  is the input 'noise'. Input can be in terms of velocity  $W_v(k)$ , or acceleration  $W_a(k)$ .

The following list of equations presents the most frequently used linear models. Each coordinate is presented by a similar form (either a Cartesian or spherical coordinate [21, 4, 63, 64]), therefore, only the x dimension is presented here.

1. 
$$\begin{bmatrix} x \\ \dot{x} \end{bmatrix} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} T/2 \\ 1 \end{bmatrix} W_{v}, \quad Q = \sigma_{v}^{2} \qquad [50, 46, 52, 64]$$
2. 
$$\begin{bmatrix} x \\ \dot{x} \end{bmatrix} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} T^{2}/2 \\ T \end{bmatrix} W_{a}, \quad Q = \sigma_{a}^{2} \qquad [25, 23, 14, 29],$$
2a. 
$$\begin{bmatrix} x \\ \dot{x} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} 1 & T & T^{2}/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \dot{x} \end{bmatrix} + \begin{bmatrix} T^{2}/2 \\ T \\ 1 \end{bmatrix} W_{a}, \quad Q = \sigma_{a}^{2} \qquad [57, 3, 17, 64]$$
3. 
$$\begin{bmatrix} x \\ \dot{x} \end{bmatrix} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} w_{1} \\ w_{2} \end{bmatrix}, \quad Q = \sigma_{a}^{2} \begin{bmatrix} T^{4}/3 & T^{3}/2 \\ T^{3}/2 & T \end{bmatrix} \qquad [38, 24, 55, 34, 4]$$
4. 
$$\begin{bmatrix} x \\ \dot{x} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} 1 & T & T^{2}/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \dot{x} \end{bmatrix} + \begin{bmatrix} T^{2}/4 \\ T/2 \\ 1 \end{bmatrix} W_{a}, \quad Q = \sigma_{a}^{2} \qquad [46, 52]$$
5. 
$$\begin{bmatrix} x \\ \dot{x} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} 1 & T & T^{2}/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \dot{x} \\ \dot{x} \end{bmatrix} + \begin{bmatrix} w_{1} \\ w_{2} \\ w_{3} \end{bmatrix}, \quad Q = A\sigma_{a}^{2} \begin{bmatrix} T^{5}/20 & T^{4}/8 & T^{3}/6 \\ T^{4}/8 & T^{3}/3 & T^{2}/2 \\ T^{3}/6 & T^{2}/2 & T \end{bmatrix} \qquad [11, 31, 16, 66]$$

6. 
$$\begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}, \quad Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma_a^2 \end{bmatrix}$$
[11, 35, 21, 62]

### 3.2 Non-Linear Equations for Target Dynamics

A non-linear model of the target dynamics is required when the state vector is in spherical coordinates [30]. Another case where a non-linear model is also used is when the state vector contains some special variables (e.g., for re-entry vehicles) [5, 8, 39, 42, 44].

The non-linear model of the target dynamics has the following general form:

$$X(k+1) = f(X(k),k) + g(X(k),k)W(k)$$
(3.3)

The non-linear target model proposed by [30] is in spherical coordinates and it has the following general form:

1a. Range channel:  

$$\begin{bmatrix} r(k+1) \\ \dot{r}(k+1) \\ w'_{r}(k+1) \end{bmatrix} = \begin{bmatrix} 1 & A & B \\ 0 & E & F \\ 0 & 0 & e^{-aT} \end{bmatrix} \begin{bmatrix} r(k) \\ \dot{r}(k) \\ w'_{r}(k) \end{bmatrix} + \begin{bmatrix} C \\ A \\ 0 \end{bmatrix} u_{r}(k) + \begin{bmatrix} D \\ G \\ (1-e^{-aT})/a \end{bmatrix} w_{r}(k)$$
1b. Bearing channel:  

$$\begin{bmatrix} b(k+1) \\ \dot{b}(k+1) \\ w'_{b}(k+1) \end{bmatrix} = \begin{bmatrix} 1 & A & B/r_{xy}(k) \\ 0 & E & F/r_{xy}(k) \\ 0 & 0 & e^{-aT} \end{bmatrix} \begin{bmatrix} b(k) \\ \dot{b}(k) \\ w'_{b}(k) \end{bmatrix} + \begin{bmatrix} C/r_{xy}(k) \\ A/r_{xy}(k) \\ 0 \end{bmatrix} u_{b}(k) + \begin{bmatrix} D/r_{xy}(k) \\ G/r_{xy}(k) \\ (1-e^{-aT})/a \end{bmatrix} w_{b}(k)$$
1c. Elevation channel:  

$$\begin{bmatrix} e(k+1) \\ \dot{e}(k+1) \\ w'_{e}(k+1) \end{bmatrix} = \begin{bmatrix} 1 & A & B/r(k) \\ 0 & E & F/r(k) \\ 0 & 0 & e^{-aT} \end{bmatrix} \begin{bmatrix} e(k) \\ \dot{e}(k) \\ w'_{e}(k) \end{bmatrix} + \begin{bmatrix} C/r(k) \\ A/r(k) \\ 0 \end{bmatrix} u_{e}(k) + \begin{bmatrix} D/r(k) \\ G/r(k) \\ (1-e^{-aT})/a \end{bmatrix} w_{e}(k)$$

where:  

$$A = (1-E)\alpha$$
,  $B = [1 + (aE - \alpha e^{-aT})/(\alpha - a)]/(\alpha a)$ ,  $C = (\alpha T - 1 + E)/\alpha^2$ ,  
 $D = [T + (aA - \alpha J)/(\alpha - a)](\alpha a)$ ,  $E = e^{-\alpha T}$ ,  $F = (e^{-aT} - E)/(\alpha - a)$ ,  
 $G = [J - A]/(\alpha - a)$ ,  $J = (1 - e^{-aT})/a$ ,

and, a = the ..., and  $\alpha =$  the ...

### 3.3 Parameters of Target Dynamics and Parametric Models

All linear models presented above can be also considered as parametric models. For example, in model 5, parameter A is the inverse of the manoeuvre's time constant and it depends on the type of manoeuvre, parameter  $\sigma_a$  depends on the target's manoeuvring capabilities and parameter T represents the radar sampling rate. Model input  $W_a$  can also be considered as parametric input [15, 11, 30, 26, 17].

Common parameter values for the above target models are the following:

- For velocity model inputs, the variance  $\sigma_v^2$  may take a value of 0.02 (m/s)<sup>2</sup> [50].
- For acceleration model inputs, the variance  $\sigma_{\alpha}^2$  may take values from 9 to 900 (ft/s<sup>2</sup>)<sup>2</sup> [23,9,23,16,30,33].
- The manoeuvre time constant A may take values from 1 to 1/60 (1/sec) [11].

### 4. RADAR EQUATIONS

Equation (2.2) represents the general form of the radar equation.

#### 4.1 Linear Equations for the Tracking Radar

When the state and the measurements are expressed at the same coordinate system, the radar measurement equation is linear with the following form :

$$Z(k) = H(k)X(k) + V(k)$$

$$(4.1)$$

where: X is the target state vector, Z is the radar measurements and V is the measurement noise. The most frequently used radar equations are expressed in the following form, one for each space coordinate.

1. 
$$Z(k) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \end{bmatrix} + V_x, \ R = \sigma_x^2$$

When the measurements are in Cartesian coordinates and the measurement noise in spherical coordinates, the two equations for X-Y (or three for X-Y-Z) coordinates are not decoupled. Therefore, a type 2 equation is used instead of two (or three) decoupled equations of type 1, i.e.:

2. 
$$\begin{bmatrix} Z_x \\ Z_y \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} X \\ \cdots \\ Y \end{bmatrix} + \begin{bmatrix} V_x \\ V_y \end{bmatrix}, R = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{bmatrix}$$
  
where:  
$$\sigma_x^2 = \sigma_r^2 \cos^2 b + r^2 \sigma_b^2 \sin^2 b, \ \sigma_y^2 = \sigma_r^2 \sin^2 b + r^2 \sigma_b^2 \cos^2 b \text{ , and,}$$
$$\sigma_{xy} = \sigma_r^2 \sin b \cos b - r^2 \sigma_b^2 \sin b \cos b \text{ , [16, 31, 25].}$$

#### 4.2 Non-Linear Equations for the Tracking Radar

The radar measurement model becomes non-linear when a Cartesian-to-spherical transformation is made by function h(.). The general form of the non-linear radar measurement equation is:

$$Z(k) = h(X(k), k) + V(k)$$
(4.2)

When the state vector variables are x, y & z and, the measurement vector variables are r, b, e, dr (range rate), and, f (Doppler frequency), the non-linear equations used by the function h(.) will be [66, 62, 59, 57, 56, 51, 49, 45, 44, 28, 26, 13, 7, 6, 3, 27, 9, 2]:

1. 
$$r(k) = \sqrt{x(k)^2 + y(k)^2 + z(k)^2} + V_r(k), \quad R = \sigma_r^2$$
  
2.  $\dot{r}(k) = \frac{x(k)\dot{x}(k) + y(k)\dot{y}(k) + z(k)\dot{z}(k)}{r(k)} + V_{r_v}(k), \quad R = \sigma_{r_v}^2$   
3.  $b(k) = \arctan\left(\frac{x(k)}{y(k)}\right) + V_b(k), \quad R = \sigma_b^2$   
4.  $e(k) = \arcsin\left(\frac{z(k)}{r(k)}\right) + V_e(k), \quad R = \sigma_e^2$   
5.  $f(k) = \left[1 - \frac{v}{c}v(t - t_0)/r\right]f + V_f(k), \quad R = \sigma_f^2$ 

### 4.3 Parameters of Radar Models and Parametric Models

The major parameters of a radar model is the sampling period T. It may take values from 0.01 to 100 sec. The other equally important parameters for the tracking radar model, are the measurement error variances i.e.,  $\sigma_x^2$ ,  $\sigma_y^2$ ,  $\sigma_{xy}^2$ ,  $\sigma_{y}^2$ ,  $\sigma_{xy}^2$ ,

### 5. FILTERING ALGORITHMS AND INITIALIZATION

A tracking filter processes the radar measurements for a target in order to achieve, at least, the following tasks:

- Reduce the measurement errors by means of time averaging
- Estimate the real velocity and acceleration of the target
- Predict the future target position or action

### 5.1 Linear and Non-Linear Filters

 $\alpha$ - $\beta$  and  $\alpha$ - $\beta$ - $\gamma$  trackers are the simplest tracking filters for estimating the position-velocity-acceleration of a target. As the means for choosing the gains  $\alpha$  and  $\beta$  become more sophisticated, the optimal  $\alpha$ - $\beta$  tracker becomes equivalent to the optimal Kalman filter. The Kalman Filter (KF) and the Lainiotis Partitioning Algorithm (LPA) are optimal linear filters and work excellent with the linear tracking models. When the tracking models are non-linear, the non-linear and sub-optimal Extended Kalman Filter (EKF) may be applied. Parametric or Multi-model cases can be resolved using the Adaptive Lainiotis Filter (ALF) or Multi-model Partitioning Filter (MMPF) [71-75].

### 5.2 Filter Initialization

All tracking filter require some initial values to start the iteration procedure. Filter initialization is an essential part of the tracking procedure. Most initial values are given a Gaussian mean and variance.

In practice, the first two or three measurements are spend to create the initial state vector X(0). The initial variance P(0/0) usually depends on the radar accuracy i.e. the measurement error variance R. e.g.:

$$X(0) = \begin{bmatrix} Z(0) \\ [Z(0) - Z(-1)]/T \\ 0 \end{bmatrix}, \quad P(0/0) = \begin{bmatrix} R & R/T & 0 \\ R/T & 2R/T^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
[11, 46]

Some times initial variance is not calculated but it is given by the designer, e.g.:

$$P(0/0) = \begin{bmatrix} 4.44 \cdot 10^{-7} & 0 & 0 & 0\\ 0 & 0.5 \cdot 10^{-6} & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1 \end{bmatrix}$$
 [9], or,  $P(0/0) = \begin{bmatrix} 10^6 I_6 & \vdots & 0\\ \cdots & \cdots & \cdots\\ 0 & \vdots & 10^2 I_3 \end{bmatrix}$  [59]

### 6. THE TTTB SIMULATOR

The above presented sets of target models (linear and non-linear), radar models (linear and non-linear) filtering algorithms (linear, non-linear and adaptive) and parameter or initial values, is collected in one common environment in order to create a general Target Tracking Test-Bed (TTTB). This test-bed can act as a common base for testing and comparing different tracking filters under the same conditions in a unified framework.

The TTTB tool is currently under development using the MATLAB application and its GUIDE tool. A user interface snapshot and a schematic of the required user steps are shown in the following figures:



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Figure 1. Typical procedure for the target tracking simulation and tracking filter application in TTTB.



Figure 2. Snapshot of the TTTB user interface for model and filter selection.

### 7. CONCLUSIONS

In this paper we presented a collection of the most often used state-space models or test cases in the target tracking problem. Subsequently, we selected and implemented the most representative of the models under a unified framework, in order to create a general purpose Target Tracking Test-Bed (TTTB). The TTTB can be used as a tracking simulator and as a common base for the test and comparison of different tracking filters. The structure of the TTTB is modular, new models and filters are easily added and their parameters can be adjusted at will. Upon its finalisation it will become a handy tool for any radar or filter specialist.

## REFERENCES

- MAYBECK P S., SUIZU R I., Adaptive Tracker Field-of-View Variation Via Multiple Model Filtering. IEEE Aer. & El. Systems Vol. AES-21 No. 4, JUL-1985.
- [2] TENNEY R R., HEBBERT R S., SANDELL N R., A. Tracking filter for Maneuvering Sources. IEEE Aut. Control Vol. AC-22 No. 2, APR-1977.
- [3] KENEFIC R J., GOULETTE P L., Sensor Netting Via the Discrete Time Extended Kalman Filter. IEEE Aer. & El. Systems Vol. AES-17 No. 4, JUL-1981.

- [4] EKSTRAND BERTIL, Analytical Steady State Solution for a Kalman Tracking Filter.' IEEE Aer. & El. Systems Vol. AES-19 No. 6, NOV-1983.
- [5] MEHRA RAMAN K. A Comparison of Several Nonlinear Filters for Reentry Vehicle Tracking. IEEE Aut. Control Vol. AC-16 No. 4, AUG-1971.
- [6] PETRIDIS V., A. Method for Bearings-Only Velocity and Position Estimation. IEEE Aut. Control VoLAC-26 No. 2, APR-1981.
- [7] WATANABE KEIGO, Application of Pseudolinear Partitioned Filter to Passive Vehicle Tracking. INT.J. SYSTEMS SCI., Vol. 15, No. 9, 1984.
- [8] CHAW-BING CHANG, WHITING R H., ATHANS M., On State & Parameter Estimation for Maneuvering Reentry Vehicles. IEEE Aut. Control Vol. AC-22 No. 1, FEB-1977.
- [9] WEISS H., MOORE J B., Improved Extended Kalman Filter Design for Passive Tracking. IEEE Aut. Control Vol. Ac-25 No. 4, AUG-1980.
- [10] CHAW-BING CHANG, TABACZYNSKI J A., Application or State Estimation to Target Tracking. IEEE Aut. Control Vol. AC-29 No. 2, FEB-1984/
- [11] SINGER R A., Estimating Optimal Tracking Filter Performance for manned Maneuvering Targets. IEEE Aer. & El. Systems Vol. AES-6 No. 4, JUL-1970.
- [12] MENDEL J M., Postflight Data Analysis by Means of Adaptive Iterated, E.K.F. IEEE Aut. Control Vol. AC-19 No. 5, OCT-1974.
- [13] NARDONE S C., AIDALA V J., Observability Criteria for Bearings-Only Target Motion Analysis. IEEE Aer. & El. Systems Vol. AES-17 No. 2, MAR-1981
- [14] BEKIR ESMAT, Adaptive Kalman Filter for Tracking Maneuvering Targets. AIAA J. Guidance. Contr. Dynamics Vol. 6 No. 5, SEP/OCT-1983,
- [15] GUPTA S N., AHN S M., Closed-Form Solutions of Target-Tracking Filters with Discrete Measurements. IEEE Aer. & El. Systems Vol. AES-19 No. 4, JUL-1983.
- [16] NAGARAJAN V. SHARMA R N., CHIDAMBARA M R., An Algorithm for Tracking a Maneuvering Target in Clutter. IEEE Aer. & El. Systems Vol. AES-20 No. 5, SEP-1984.
- [17] KALATA PAUL R., The Tracking Index: A Generalized Parameter for a-b and a-b-c Target Trackers. IEEE Aer. & El. Systems Vol. AES-20 No. 2, MAR-1984.
- [18] TANG Y M., BORRIE J A., Missile Guidance Based on Kalman Filter Estimation of Target Maneuver. IEEE Aer. & El. Systems Vol. AES-20 No. 6, NOV-1984.
- [19] LEFAS C C., Using Roll-Angle Measurements to Track Aircraft Maneuvers. IEEE Aer. & El. Systems Vol. AES-20 No. 6, NOV-1984.
- [20] LEFAS C C., Algorithms for Improved. Heading Assisted. Maneuver Tracking. IEEE Aer. & El. Systems Vol. AES-21 No. 3, MAY-1985.
- [21] SINGER R A., BEHNKE K., Real-Time Tracking filter Evaluation and Selection for Tactical applications. IEEE Aer. & El. Systems Vol. AES-7 No. 1, JAN-1971.
- [22] SPINGARN K., WEIDEMANN H L., Linear Regression Filtering and Prediction for Tracking Maneuvering Aircraft Targets. IEEE Aer. & El. Systems Vol. AES-8 No. 6, NOV-1972.
- [23] FRIEDLAND B., Optimum Steady-State Position and Velocity Estimation Using Noisy Sampled Position Data IEEE Aer. & El. Systems Vol. AES-9 No. 6, NOV-1973.
- [24] SINGER R A., SEA R G., HOUSEWRIGHT K B., Derivation and Evaluation of Improved Tracking Filters for Use in Dense Multitarget Environments. IEEE Inform. Theory Vol. IT-20 No74, JUL-1974.
- [25] TRUNK G V., WILSON J D., Tracking Filters for Multiple-Platform Radar Integration. NRL-Report 8087, DEC-1976
- [26] GHOLSON N H., MOOSE R L., Maneuvering Target Tracking Using Adaptive State Estimation. IEEE Aer. &. El. Systems Vol. AES-13 No. 3, MAY-1977.
- [27] RICKER G G., WILLIAMS J R., Adaptive Tracking Filter for Maneuvering Targets. IEEE Aer. & El. Systems Vol. AES-14 No. 1, JAN-1978.
- [28] LINDGREN A G., GONG K F., Position and Velocity Estimation Via Bearing Observations. IEEE Aer. & El. Systems Vol. AES-14 No. 4, JUL-1978.
- [29] CHAN Y T., HU A G C., PLANT J B. A. Kalman Filter Based Tracking Scheme with Input Estimation. IEEE Aer. & El. Systems Vol. AES-15 No. 2, MAR-1979.
- [30] MOOSE R L., VANLANDINGHAM, McCABE, Modeling and Estimation for Tracking Maneuvering Targets. IEEE Aer. & El. Systems Vol. AES-15 No. 3, MAY-1979.
- [31] FARINA A., PARDINI S., Multiradar Tracking System Using Radial Velocity Measurements. IEEE Aer. & El. Systems Vol. AES-15 No. 4, JUL-1979.
- [32] FARUQI F A., DAVIS R C., Kalman Filter Design for Target Tracking. IEEE Aer. & El. Systems Vol. AES-16 No. 3, JUL-1980.
- [33] FITZGERALD R J., Simple Tracking Filters: Steady-State Filtering and Smoothing Performance. IEEE Aer. & El. Systems Vol. AES-16 No. 6, NOV-1980.
- [34] CASTELLA F R. An Adaptive Two-Dimensional Kalman Tracking Filter. IEEE Aer. & El. Systems Vol. AES-16 No. 6, NOV-1980.
- [35] EILTS H S., A Sampled Data Delay-Lock Loop Implemented as a Kalman Predictor. IEEE Aer. & IT. Systems Vol. AES-16 No. 6, NOV-1980.
- [36] KENDRICK J D, MAYBECK P S., REID J G., Estimation of Aircraft Target Motion Using Orientation Measurements. IEEE Aer. & El. Systems Vol. AES-17 No. 2, MAR-1981.
- [37] MAYBECK P S., JENSEN R L., HARNLY D A., An Adaptive Extended Kalman Filter for Target Image Tracking. IEEE Aer. & El. Systems Vol. AES-17 No. 2, MAR-1981.
- [38] CASTELLA F R., Tracking Accuracies with Position and Rate Measurements. IEEE Aer. & El. Systems Vol. AES-17 No. 3, MAY-1981.
- [39] ALSPACH D L., A Gaussian Sum Approach to the Multi-Target Identification-Tracking Problem. AUTOMATICA Vol. 11, -1975.
- [40] FITZGERALD R J., Simple Tracking Filters: Closed-Form Solutions. IEEE Aer. & El. Systems Vol. AES-17 No. 6, NOV-1981.
- [41] SINGER R A., SEA R G., New Results in Optimizing Surveillance System Tracking and Data Correlation Performance in Dense Multitarget Environments. IEEE Aut. Control Vol. AC-18 No. 6, DEC-1973.
- [42] BAR-SHALOM Y., TSE E., Tracking is a Cluttered Environment With Probabilistic Data Association. AUTOMATICA Vol. 11, -1975.
- [43] BAR-SHALOM Y., Tracking Methods in a Multitarget Environment (Survey). IEEE Aut. Control Vol. AC-23 No. 4, AUG-1978.
- [44] MILLER K S., LESKIW DM. Nonlinear Estimation with Radar Observations. IEEE Aer. & El. Systems Vol. AES-18 No. 2, MAR-1982.
- [45] AIDALA V J., NARDONE S C. Biased Estimation Properties of the Pseudo-linear Tracking Filter. IEEE Aer. & El. Systems Vol. AES-18 No. 4, JUL-1982.
- [46] BAR-SHALOM Y., BIRMIWAL K., Variable Dimension Filter for Maneuvering Target Tracking. IEEE Aer. & El. Systems Vol.

AES-18 No. 5, SEP-1982.

- [47] FITZGERALD R J. Simple Tracking Filters: Position and Velocity Measurements. IEEE Aer. & El. Systems Vol. AES-18 No. 5, SEP-1982.
- [48] FARINA A., HANLE E., Position Accuracy in Netted Monostatic and Bistatic Radar. IEEE Aer. & El. Systems Vol. AES-19 No. 4, JUL-1983.
- [49] AIDALA V J. Kalman Filter Behavior in Bearings-Only Tracking Applications. IEEE Aer. & El. Systems Vol. AES-15 No. 1, JAN-1979.
- [50] KUO-CHU CHANG, BAR-SHALOM Y., Joint Probabilistic Data Association for Multitarget Tracking with Possibly Unresolved Measurements and Maneuvers. IEEE Aut. Control Vol. AC-29 No. 7, JUL-1984.
- [51] NARDONE S C. LINDGREN A G., GONG K F., Fundamental Properties and Performance of Conventional Bearings-Only Target Motion Analysis. IEEE Aut. Control Vol. AC-29 No. 9, SEP-1984.
- [52] BIRMIWAL K., BAR-SHALOM Y., On Tracking & Maneuvering Target in Clutter. IEEE Aer. & El. Systems Vol. AES-20 No. 5, SEP-1984.
- [53] SINGER R A. KANYUCK A J., Computer Control of Multiple Site Track Correlation. AUTOMATICA Vol. 7, -1971,
- [54] Von G. van KEUK, Ein Verallgemeinertes Kalman- filter zur Triangulierung in Einem Variablen nulti-sensor-System. REGELUNGSTECHNIK, Heft 10, OCT-1978.
- [55] YU M H., MEYER M P., Closed-Form Solution of a Recursive Tracking Filter with A Priori Velocity Initialization. IEEE Aer. & El. Systems Vol. AES-21 No. 2, MAR-1985.
- [56] HAMMEL S E., AIDALA V J., Observability Requirements for Three-Dimensional Tracking via Angle Measurements. IEEE Aer. & El. Systems Vol. AES-21 No. 2, MAR-1985.

[57] YANG Z., Particle Velocity Estimation Using Kalman Filtering. IEEE Aer. & El. Systems Vol. AES-21 No. 3, MAY-1985.

- [58] MOOSE R L., DAILEY T E., Adaptive Underwater Target Tracking Using Passive Multipath Time-Delay Measurements. IEEE Ac. Sp. & Sign. Processing Vol. ASSP-33 No. 4, AUG-1985.
- [59] SONG T L., SPEYER J L., A Stochastic Analysis of a Modified Gain Extended Kalman Filter with Applications to Estimation with Bearings Only Measurements. IEEE Aut. Control Vol. AC-30 No. 10, OCT-1985.
- [60] FITZGERALD R J., Track Biases and Coalescence with Probabilistic Data Association. IEEE Aer. & El. Systems Vol. AES-21 No. 6, NOV-1985.
- [61] LINDGREN A G., IRZA J., NARDONE S C., Trajectory Estimation with Uncertain and Nonassociated Data. IEEE Aer. & El. Systems Vol. AES-22 No. 1, JAN-1986.
- [62] BAHETI R S., Efficient Approximation of Kalman Filter for Target Tracking. IEEE Aer. & El. Systems Vol. AES-22 No. 1, JAN-1986.
- [63] MCAULAY R J., DENLINGER E., A Decision- Directed Adaptive Tracker. IEEE Aer. & El. Systems Vol. AES-9 No. 2, MAR-1973.
- [64] SMITH P., BUECHLER G., A. Branching Algorithm for Discriminating and Tracking Multiple Objects. IEEE Aut. Control Vol. AC-20 No. 1, FEB-1975;
- [65] TRUNK G y., WILSON J D., Track Initiation of Occasionally Unresolved Radar Targets. IEEE Aer. & E1.Systems Vol. AES-17 No7l, JAN-1981.
- [66] FARINA A., PARDINI S., Track-While-Scan Algorithm in a Clutter Environment. IEEE Aer. & El. Systems Vol. AES-14 No. 5, SEP-1978.
- [67] THOMAS H W., Maneuver Handling in & Multiradar. A. T. C. System. IEE PROCEEDINGS Vol. 126 No. 6, JUN-1979.
- [68] FARINA A., PARDINI S., Survey of Radar Data-Processing Techniques in Air-Traffic-Control and Surveillance Systems. IEE PROCEEDINGS Vol. 127 Pt.F No. 3, JUN-1980.
- [69] MAYBECK P S., Advanced Applications of Kalman Filters and Nonlinear Estimators in Aerospace Systems. CONTROL & DYNAMIC SYSTEMS (LEONDES Ed.) Vol. 20, -1983.
- [70] FARINA A., Multistatic Tracking and Comparison with Netted Monostatic Systems. RADAR-82 Proc. Int. Conf. London, OCT-1982.
- [71] D. G. Lainiotis and V. C. Moussas, "Adaptive Filtering Algorithms and Target Tracking", in *Proceedings of the AFCET-IASTED International Symposium on Identification and Pattern Recognition IPAR'86, Univ. Paul Sabatier, Toulouse, France*, Vol. III, p. 337-360, June 18-20, 1986.
- [72] S. K. Katsikas, D. G. Lainiotis and V. C. Moussas, "Lainiotis filters application to sonar and radar tracking: A survey", in *Proceedings of the IFAC Workshop on Expert Systems and Signal Processing in Marine Automation, Lyngby, Denmark*, pp. 303-310, Aug. 1989.
- [73] Katsikas, S. K., Leros, A. K., Lainiotis, D. G., (1994), 'Passive tracking of a maneuvering target: an adaptive approach', Signal Processing, IEEE Trans on, vol 42, Issue 7, pp. 1820-1825.
  [74] Nikitakos, N. V., Leros, A. K., Katsikas, S. K., (1998), 'Towed array shape estimation using multi-model partitioning filters', Oceanic
- [74] Nikitakos, N. V., Leros, A. K., Katsikas, S. K., (1998), 'Towed array shape estimation using multi-model partitioning filters', Oceanic Engineering, IEEE Journal of, vol 23, Issue 4, pp. 380-384.
- [75] Moussas, V. C., Likothanassis, S. D., Katsikas, S. K., Leros, A.K., (2005), 'Adaptive On-Line Multiple Source Detection' Acoustics, Speech, and Signal Processing, Proceedings, (ICASSP '05). IEEE International Conference on Volume 4, March 18-23, pp. 1029-1032.