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A general method for raw fatigue crack growth data processing and structural reliability assessment

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ABSTRACT: In this work a non-linear method for the modelling and identification of the fatigue crack growth (FCG) is presented. Unlike the standard method (e.g. ASTM E647-93) which is suitable mainly for Paris-like FCG relationships and requires transformations of the data (determination of the derivative $d\alpha/dN$, logarithms etc.) the proposed method is general and applicable to any FCG functional form. Its great advantage is also that it can treat the α vs. N raw data directly. The method is based on the powerful and well-known non linear least squares techniques for the parameter estimation. The relevant algorithms and its computational efficiency are also fully developed and proved. A case study with the Virkler data is performed. Several FCG laws are examined and the present non-linear as well as linear methodologies are implemented and compared, where possible. As is shown, the new parameter estimates, in combination with Monte-Carlo numerical simulations, provide realistic predictions of the crack length distributions and generally lead to improved structural reliability assessments.

1 INTRODUCTION

Considerable research effort has been devoted on the modelling of the fatigue crack propagation (FCG) using a linear elastic fracture mechanics (LEFM) approach (Sobczyk 1992, Hoeppner 1974, Fuchs 1980). The developed models, mainly semi-empirical, allow the prediction of the behaviour of the crack α as a function of cycles N. The material characteristics, the dimensions of the specimen, the load and other conditions appear as parameters of the FCG models. As is well known however, the experimental data of FCG, required for the parameter estimation, show a considerable scatter even if they are collected from very well controlled experiments. In order to compensate this scatter, some parameters are randomised and the distributions of their values are used in the place of the fixed values. These probabilistic or stochastic versions of the FCG models are used in prediction, to determine either a distribution of the crack size α at a given number of cycles N, or, a distribution of cycles N needed to reach a crack size α.

The stochastic FCG models constitute an essential part of the probabilistic structural reliability assessment (Lucia 1985, Yao 1986, Ditlevsen 1986), which is performed using advanced codes (COVASTOL, RELIEF, Monte-Carlo etc.) (Elishakoff 1983, Dusfresne 1985, Stavrakakis 1990), developed to predict the crack propagation and to estimate the life-time distribution under fatigue loading. It is thus clear that the quality of the prediction depends directly on the quality of the method used to process the experimental data and to estimate the parameters. A poor estimation of the parameters will lead to an inaccurate prediction of the life-time, even if a sophisticated prediction code is used.

The currently used standard method for this estimation, ASTM E647-93, presents several weak points. First, the method is strictly related to the application of Paris-like models to describe the FCG. Then, the determination of the derivative $d\alpha/dN$, required by this method, introduces a scatter in the FCG rate data which varies considerably with the data processing technique used to evaluate the raw test data. Specifically, the variability associated with the incremental polynomial and total polynomial techniques is substantially lower than that associated with the modified difference and secant techniques (Clark 1975). The incremental polynomial method is the one suggested by the ASTM E647-93 standard. Virkler et al.(Virkler 1979) in the analysis of their FCG data, using the same processing techniques with

some minor modifications, have concluded differently. They found that the polynomial methods perform poorly because their inherent smoothing action disguises the actual crack propagation behaviour, especially if the data are observed from a non-macroscopic viewpoint, and that the finite difference method describes more accurately the irregularities and inconsistencies of the raw data and in effect describes quantitatively the variability of the crack propagation process. Another problem of the standard method is that, even after the transformations have been performed on the α vs. N data (derivative & logarithm), the resulting $\log(d\alpha/dN)$ vs. $\log(\Delta K)$ data are still non-linear. This non-linear relationship poses limitations for the least squares regression technique which is used to obtain a single-value index of variability.

The need to introduce more advanced or alternative estimation and identification methods (Solomos 1991) is thus obvious. The undesirable subjectivity of the analytical technique used to convert the raw test results to growth rate data, as well as, the limitation to use simple laws such as the Paris law for FCG rate modelling, may be alleviated if new methods are used, based on the current state-of-the-art of iterative minimisation methods for non-linear regression. To satisfy those needs, this paper presents a method for the identification of the model parameters, capable of determining the parameters of any FCG model. The proposed method uses well established procedures in other engineering fields based on non-linear modelling (Jaszwinski 1970, Astrom 1971). Its performance is compared to the standard method, using the Paris law and experimental FCG data. Predictions of the crack size distributions, performed by the Monte-Carlo simulations, are also used to compare the results of the two methods.

2 IDENTIFICATION OF THE FCG MODEL

Several relationships have been proposed for the modelling of the fatigue crack propagation (Schutz 1979, Thoft-Christensen 1982, Yang 1983, Lin 1985). The models vary from simple to more complex ones (e.g. the hyperbolic sine model):

$$\alpha = A\, e^{C_1 N} \qquad (Shanley) \qquad (2.1)$$

$$\frac{d\alpha}{dN} = 10^{C_1 \cdot \sinh[C_2 \cdot (\log \Delta K + C_3)] + C_4} \qquad (Larsen) \qquad (2.2)$$

The most widely used models, however, are the Paris-Erdogan equation and its variations:

$$\frac{d\alpha}{dN} = C\,(\Delta K)^n \qquad (Paris) \qquad (2.3)$$

$$\frac{d\alpha}{dN} = \frac{C(\Delta K)^n}{(1-R)K_C - \Delta K} \qquad (Forman) \qquad (2.4)$$

Thus it can be generally agreed that the FCG can be described by some equation of the following form

$$\frac{d\alpha}{dN} = f(\Delta K, \beta), \text{ or, } \alpha = g(\alpha, \Delta K, \beta, N) \qquad (2.5)$$

where ΔK is the stress intensity factor range and β is a vector of appropriate material parameters, generally related among them. In the case of Paris model the parameter vector is $\beta^T = [C \;\; n]$. This is also the case for the Forman's model when R (stress ratio) and K_c (fatigue toughness) are known. The Shanley's model has $\beta = C_1$ and the Larsen's model has a larger parameter vector $\beta^T = [C_1\; C_2\; C_3\; C_4]$. A sensitivity analysis of a model can indicate which parameters may be considered as constants and which must be randomised, in order to reduce the model's complexity. The estimation of the parameter vector β can be done by applying identification methods on the FCG experimental data as described below.

2.1 *Linear regression (standard method)*

When using the standard method to estimate the model's parameters, the choice of the FCG model is restricted to the Paris law (2.3) and its extensions. From the two unknown parameters C and n, the n may be considered as fixed and only the C is randomised. The procedure starts by calculating, for every experimental value of α, the corresponding value of the stress intensity factor range ΔK from the general equation $\Delta K = \Delta\sigma\sqrt{\pi\alpha}\, Y(\alpha)$, where $\Delta\sigma$ is the stress range and $Y(\alpha)$ is the configuration correction factor. For the case under study below (Virkler's center cracked panels), $Y(\alpha)$ can be sufficiently approximated by the closed form expression

$$Y(\alpha) = \sqrt{1/\cos(\pi\alpha/w)} \qquad (2.6)$$

Then, the derivative $d\alpha/dN$ of the experimental function $\alpha = f(N)$, is calculated from the α vs. N data. From the various proposed methods (graphical procedure, secant method, modified difference method, incremental polynomial method, total polynomial method, etc.) the secant and the incremental polynomial are mainly used. The

transformation of the experimental data ends by taking the logarithms $\log(d\alpha/dN)$ and $\log(\Delta K)$. The transformation gives, for example, the following linear form to the Paris equation:

$$\log\left(\frac{d\alpha}{dN}\right) = \log C + n \cdot \log \Delta K \tag{2.7}$$

Clearly, n is the slope and $\log C$ the intercept of a straight line.

As is usually the case, it becomes evident from the form of the plot $\log(d\alpha/dN)$ vs. $\log(\Delta K)$ that the Paris law cannot fit the whole set of points satisfactorily. The data must be split into smaller regions of ΔK, in order to be able to describe them using the Paris equation. Usually they are divided in three parts, in each of which the points exhibit an almost linear behaviour. For every such ΔK region a pair of parameter values (C, n) must be defined. However even within these regions, due to inherent material variability, conditions inaccuracies, and, measure-ment and calculation errors, the data points will not lie on a straight line. This variability can be represented by an error term $\log e$ in the Paris equation:

$$\log\left(\frac{d\alpha}{dN}\right) = \log C + n \cdot \log \Delta K + \log e \tag{2.8}$$

The parameters C and n, as well as the standard deviation of the error term $\log e$ are determined by the method of linear least-squares. Regression lines are obtained using this procedure for every ΔK region. Usually the crack growth data are considered as one group, for the determination of the fixed parameter n of each ΔK region, and as separate tests, for the determination of the randomised parameter C of the same region. The standard method concludes by reporting a value for the parameter n and a value distribution for the parameter C, for every ΔK region.

2.2 *Generalised Linear Least Squares*

The standard method applies the linear least squares to fit a set of data to a straight line. An immediate generalisation is to fit the data points (x_i, y_i) to a model which is not just a linear combination of 1 and x (namely $a+bx$), but rather a linear combination of any M specified functions of x. For example these functions could be $1, x, x^2, x^3, \ldots, x^{M-1}$, in which case their general linear combination,

$$y(x) = a_1 + a_2 x + a_3 x^2 + \ldots + a_M x^{M-1} \tag{2.9}$$

is a polynomial of degree M-1. These functions could equally be sines, cosines or logarithms. The general form of this kind of model is:

$$y(x) = \sum_{k=1}^{M} a_k X_k(x) \tag{2.10}$$

where $X_1(x), \ldots, X_M(x)$ are arbitrary fixed functions of x. The functions $X_k(x)$ can be strongly non-linear functions of x, as, the term "linear" only refers to the model's dependence on its parameters (Astrom 1971).

The so-called "general linear least squares" may be used to fit more complex FCG models. Several transformations may take place in order to give the model the desired form. For example, if equation (2.4) is the model, then, by taking logarithms and replacing $\log(d\alpha/dN)$ by $y(x)$ and $\log(\Delta K)$ by x, we have:

$$y(x) = \log C + n \cdot x - \log\left[(1-R) \cdot K_c - e^x\right] \tag{2.11}$$

This generalised procedure is still limited by the model's linear dependence on the parameters. In order to fit more complex and highly non linear models, a general non-linear method is needed.

2.3 *A general non-linear method (non-linear least squares)*

In this method the estimation of the parameters is obtained using the non-linear least-squares. The algorithm used has been originally proposed by Marquardt (1963) for the least-squares estimation of non-linear parameters. This technique is developed for the identification of non-linear systems. When considering these systems, several specific assumptions are made concerning the model structure. It is usually assumed that the system equations are known except for a number of parameters. In a typical case the identification problem can be formulated as follows:

The model of the system is described by the state equations in x and y

$$\frac{dx}{dt} = f(x, u, \beta, t)$$

$$y_m = g(x, u, \beta, t) \tag{2.12}$$

where β is the parameter vector, and u is the input. The criterion for the parameter estimation is provided by the loss function

$$V(y, y_m) = V(\beta) = \int_0^T \left[y(k) - y_m(k, \beta)\right]^2 dk \tag{2.13}$$

where y is the process output and y_m the model output for instance k, and T the observation time of the phenomenon under consideration. The estimation problem for a non-linear parametric model thus reduces to a non-linear optimisation problem. The

Table 1. Capabilities of the parameter estimation methods.

Type of the Model's Dependence on:		Identification Procedure Capable to Solve the Problem		
Parameters	State (x)	Linear Regression	Generalised Least Squares	Non-Linear Least Squares
Linear	Linear	Yes	Yes	Yes
Linear	Non-linear	No	Yes	Yes
Non-linear	Non-linear	No	No	Yes

necessary computer procedures for the above technique can be found in any scientific computer library such as IMSL, NAG, SAS, Numerical Recipes, etc. For the present work, the non-linear modelling procedures of the SAS, SAS/ETS and Numerical Recipes packages were used.

Before applying the general method, a FCG model must be selected. It is possible to use a Paris-like model equation (2.3), as in the standard method, or, any other FCG model of interest e.g., (2.1), (2.2), (2.4) or (2.5). Once selected, the model equation must first be written in a form compatible with the FCG data in hand and then introduced in the non-linear modelling procedure. Usually, an integrated or recursive form of the model such as

$$\alpha = f(N), \text{ or, } N = f(\alpha), \text{ or,}$$
$$\alpha_{k+1} = G(\alpha_k, \Delta K, \beta, N_k, N_{k+1}), \text{ etc.} \qquad (2.14)$$

is in accordance with the discrete iterative identification algorithms, developed on digital computers, and thus suitable to treat the discrete FCG data.

When the model is ready the modelling procedure may be applied to estimate the values of the unknown parameters for every experiment and, if required, for every ΔK region. As a result, value distributions for all the model parameters are available. If these distributions or the analysis of the model show that some of the parameters can be fixed, then the estimation can be repeated using these fixed values.

2.4 *Discussion*

The proposed method uses directly the original (α vs. N) raw FCG data and avoids any transformation that can affect their precision. The standard method instead, in an attempt to give a linear form to the data, applies two non linear transformations. Moreover, the scatter in the FCG rate data varies considerably with the data processing technique used to evaluate the raw test data. This is even more important as, the selection of the best technique (polynomial, secant, etc.), is not quite clear, due to contradictory opinions found in the literature (ASTM, Virkler). By applying the $\log_{10}$ transformation, the FCG rate data are taking a less non-linear form, which may permit the approximation of the data with the linear Paris form (2.8).

When the identification is applied, what happens in fact is, that the proposed general method fits the original non-linear FCG data with a non-linear function of our choice and estimates any unknown parameter. The standard method, instead, fits the less (but still) non-linear and error corrupted FCG data with a linear function and estimates its slope and interception. It is also clear that the standard method may compete with the general non-linear method only for a certain number of problems, where only the use of a Paris-like law is required (Table 1).

3 COMPARISON OF THE METHODS USING EXPERIMENTAL DATA

For assessing the methods with real data, the experimental data from D.A. Virkler et al.[11] on aluminium (2024-T3) test specimens were used. These crack growth data were generated from 2.54 mm thick centre-cracked panels, 558.8 long and 152.4 wide. 68 replicate tests were performed under identical loading conditions ($\Delta\sigma$ = 48.263 kPa). Crack growth was monitored with a microscope of a 0.001 mm resolution. The data were recorded at consistent discrete crack length levels, starting at a half crack length α of 9.00 mm and extending to a final length of 49.80 mm. The measurement interval $\Delta\alpha$ was 0.20 mm first, and it was increased to 0.40 mm and to 0.80 mm, after half crack lengths of 36.20 mm and 44.20 mm were reached, respectively.

3.1 *Estimation Results*

Following the standard procedure, the derivative

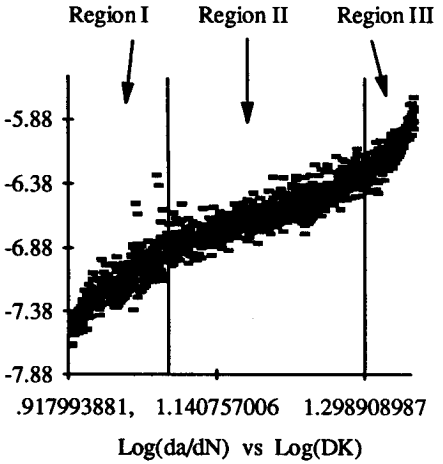

Figure 1. Part of the Virkler's data illustrating the three ΔK regions.

Table 2. C & n estimations (linear regression).

Group of data:	Region I	Region II	Region III	Entire Curve
E[log$C$]	-25.289	-21.982	-26.605	-23.13847
E[n]	3.79937	2.44886	3.98811	2.87150

Table 3. C & n estimations (non-linear method).

Group of data:	Region I	Region II	Region III	Entire Curve
E[log$C$]	-25.289	-21.982	-26.605	-23.13849
E[n]	3.79938	2.44886	3.98811	2.87151
Var(n)	0.320	0.132	1.175	0.0273

$d\alpha/dN$ and the values of ΔK were calculated from the experimental data. The derivative $d\alpha/dN$ was calculated using the secant method. After taking the logarithms of both quantities, the resulting log($d\alpha/dN$) vs. log(ΔK) curves were plotted (Figure 1). The plot may be separated in the following three ΔK regions, before applying the linear least-squares. Region I contains all points where $\Delta K \leq 12.04$ (in N/mm$^{3/2}$) or $\alpha \leq 18.4$ mm, region II : $12.04 < \Delta K \leq 20.28$ or $18.4 < \alpha \leq 39.0$ mm, and, region III : $\Delta K > 20.28$ or $\alpha > 39.0$ mm.

During the linear regression, the data were first considered as one group (instead of 68) and they were used to estimate the slope n and intercept C of the straight line. The estimated values of n & C, for every ΔK region, are shown in Table 2. Had all the data been considered simultaneously in one set, the linear regression method would have produced the results of the last column of Table 2.

The estimation was repeated for the parameter C only, by fixing the value of the parameter n. The estimated distributions of the parameter C for every ΔK region are shown in Figure 2.

The same form of the Paris equation is also used by the non-linear procedure. For comparison reasons the estimation of the parameters is repeated for all ΔK regions, as they were defined for the standard method. The non-linear treatment of the data as 68 curves produces the results shown on Table 3.

Again, the non-linear treatment of the data as one group of points, instead of 68, produces the same results for the three ΔK regions, but, slightly different results when processing the entire curve, namely: E[log$C$] = -23.193, E[n] = 2.8961, Var(n) = 0.0771.

The estimation was repeated for the parameter C only, by fixing the value of the parameter n. The estimated distributions of C are identical to those presented in Figure 2.

The results of the two methods (Tables 2 & 3) are almost identical. They show some differences after the fifth decimal place, mainly due to round-off errors and different algorithmic structure. It is worth comparing the above results with those derived by employing the maximum likelihood method (Ditlevsen 1986), i.e. E[log$C$]= -26.18, E[n]= 2.872, Var(n)= 0.02898.

Next, the more complex task of fitting equation (2.2) to the Virkler data is undertaken. Obviously, the standard method can not be of any help, and resorting to the general non-linear approach is necessary. After performing some preliminary runs for assessing the parameters sensitivity it is concluded that the dominant parameter C_1 can be practically considered as constant, equal to C_1= 0.5. The other parameters are then estimated, first by considering the data as 68 different curves and then by considering all data points as one group. The results of the parameter estimation were: E[C_2]= 4.8777, E[C_3]= -1.1843 & E[C_4]= -6.6464, for the first, and, E[C_2]= 3.627, E[C_3]= -1.4022 & E[C_4]= -6.1633, for the second case, respectively.

These results compare well with those where 64 of the 68 experiments and the method of maximum likelihood had been used (Lin 1985): C_1= 0.5, C_2= 3.4477, C_3= -1.3902, C_4= -4.5348.

The estimation was repeated for each one of the parameters, by fixing the values of the remaining three. The distributions of two parameters C_3 & C_4 are shown in Figure 3.

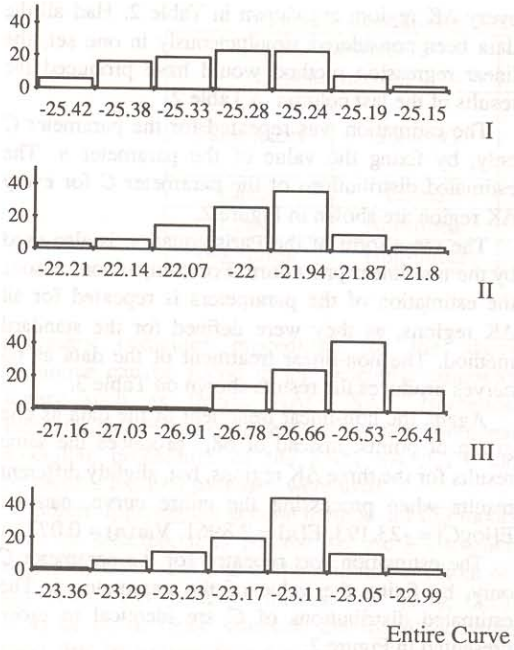

Figure 2. C distributions (% vs logC).

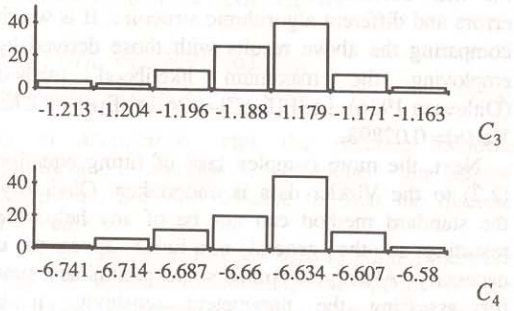

Figure 3. C_3 & C_4 distributions (% vs. value).

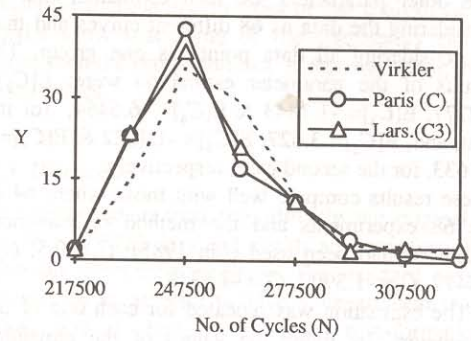

Figure 4. Distributions of N at the final crack size.

3.2 *Simulation*

The results of the non-linear approach are next inserted correspondingly in equations (2.2) and (2.3), and Monte-Carlo simulations are carried out. The values of the randomised parameters - C for eq. (2.2) and C_3 for eq. (2.3) respectively - have been created using the above histograms and a uniform random number generator. For improved results, specialised random number generators may be used to fit better the estimated distributions .

After 200 Monte-Carlo runs, the distributions of cycles (N) to reach the final crack size α, are plotted in figures 4 & 5, together with the corresponding data from Virkler, and as is observed they show the proper characteristics of the original data.

4. CONCLUSIONS

A general method for processing fatigue crack propagation data has been presented, in an attempt to overcome the restrictions of the standard procedures. The results from the parameter estimation and simulation prove that the combined application of non-linear modelling and Monte-Carlo simulation, creates a powerful general method for FCG data processing and structural reliability assessment. Both techniques are general i.e. independent of the type of the FCG law and of the form of the data. Moreover, they are computationally robust and they have been applied successfully with no problems for the cases under consideration.

The advantages of this general method become clear in the most difficult cases i.e. when selecting the best model among others of different type, or, when the modelling equation is highly parametric and non-linear. In order to create a more complete tool, the presented general method could also include a third technique for real-time estimation and identification. This technique comes from the areas of non-linear filtering and is beyond the scope of the present work.

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