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A TIME SERIES APPROACH TO FATIGUE CRACK PROPAGATION

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ABSTRACT

The inherently nonlinear phenomenon of fatigue crack propagation is modeled as a linear random process. To a first approximation, simple, nonstationary time series models are introduced and standard techniques for determining the parameters of autoregressive integrated moving-average processes are applied. Multiplicative time series models are next utilised for the representation of a group of crack history curves. Implementation of the models on the Virkler experimental data set yields satisfactory results. Reliable Gaussian approximations to the distribution of the time required by a crack to reach a specified critical length are obtained, and the usefulness of the approach is demonstrated when updating lifetime predictions after periodic inspections.

1. INTRODUCTION

As fatigue is an important factor in the engineering design, considerable effort has been devoted into developing analytical methods for its modeling [1,2]. Fatigue in metals is manifested with the appearance of small cracks, which, under alternating loading, propagate and eventually lead to failure of a component or a structure. It is evident that a flawed structure is not necessarily disqualified from service as it may still possess substantial remaining life. The assessment of the rate of growth of a crack is thus important for the avoidance of a catastrophic failure.

It is also widely accepted that the fatigue crack propagation phenomenon is probabilistic in nature. Several sets of experimental results can confirm this claim. Figure 1, for example, shows the crack propagation time histories of identical specimens under identical loading conditions [3].

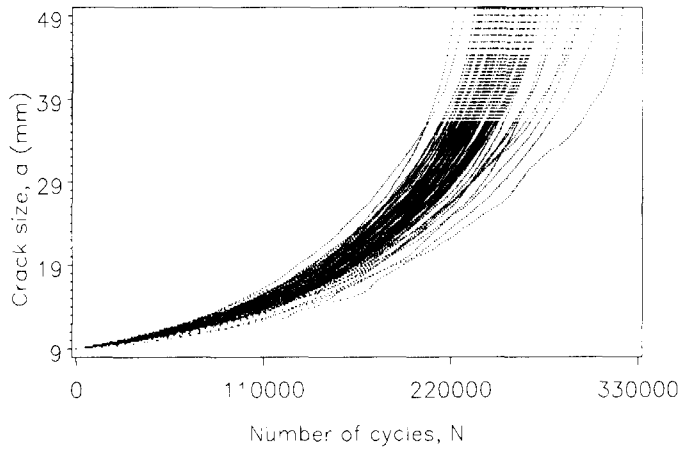

Fig. 1. Crack growth histories of center cracked specimens, Ref. [3].

The scatter of these curves is noticeable. It thus becomes clear that for assessing effectively the reliability of a cracked component, one has to resort to probabilistic and statistical methods. In such an assessment one would need to primarily determine the probability distribution of the damage (crack size) at some future time, and the probability distribution of the time required by a crack to reach a specified critical size.

Several methods with varying degrees of success have been developed so far to these ends. Prominent among them are those based on the Markov processes approach [4–11]. Going a step further, it is the objective of the present work to investigate on the possibility of applying the theory of linear times series in this field. It is felt that if such an approach proves fruitful, the fully developed techniques of time series theory will offer a powerful tool to both the fatigue researcher and the designer.

As the analyses to be used are data based, the experimental results of Ref. [3], shown in Fig. 1, have been selected for consideration. The specimens used to obtain these results were center cracked panels of 2024-T3 aluminum alloy 2.54 mm thick, 558.8 mm long and 152.4 mm wide. A central stress raiser slit 2.54 mm long was machined by electro-discharging and the center line of the specimen was used as measuring reference. The crack initiation was accomplished with an initially higher load level. The total number of specimens was 68. The load was of sinusoidal form with frequency 20 Hz, maximum value $P_{\max} = 23.353$ kN and load ratio $R = 0.20$. Data recording started at half crack length, a, of 9 mm and extended to a final length of 49.80 mm. The accumulated number of cycles were recorded for each $\Delta a = 0.20$ mm of crack growth over the range $9.00 \leqslant a \leqslant 36.20$ mm, for $\Delta a = 0.40$ mm over the range $36.20 \leqslant a \leqslant 44.20$ mm and for $\Delta a = 0.80$ mm over the range $44.20 \leqslant a \leqslant 49.80$ mm. Thus each specimen produced 165 data points.

Clearly, due to the mode of recording, these data are not equidistant in time (= number of cycles) as the relevant analysis requires. With the objective of preserving the raw data, the way chosen to overcome this minor problem is by switching the roles of the variables, i.e. by considering the crack size a as the independent variable ("time") and the number of cycles N as the dependent variable whose behavior is to be studied. This point of view is in full accord with the Ref. [12] critical analysis of fatigue crack growth laws, and also with Ref. [13], where it is shown that the number of cycles variable lends itself naturally to be modeled as a random

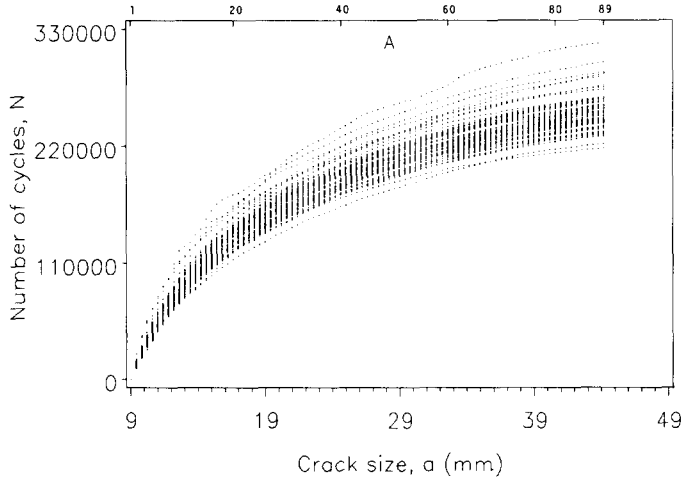

Fig. 2. Modified presentation of crack growth histories of Ref. [3].

process. Next, in order to avoid overcongested and overdetailed data, in which local anomalies might disturb overall trends, data pairs (N, a) corresponding to $\Delta a = 0.40$ mm and starting from $a_0 = 9$ mm are retained for each replicate. For this reason, the seven last points of each replicate, corresponding to $\Delta a = 0.80$ mm, are excluded as non conforming. It is believed that they also do not carry any additional information on the fatigue crack growth (FCG) investigation, as by that point all relevant characteristics of the curves have been manifested.

Thus, finally, the working data base contains 68 replicates with 89 points each, sampled at intervals $\Delta a = 0.40$ mm, starting at $a_0 = 9$ mm and terminating at $a_f = 44.2$ mm, Fig. 2. As shown in this figure, for referencing and computational convenience, the crack sizes will also be identified, indiscriminately, by their cardinal number A, i.e. $A = 1$ corresponds to $a = 9$ mm, $A = 2$ to $a = 9.4$ mm, ..., $A = 89$ to $a = 44.2$ mm.

2. THEORETICAL BACKGROUND

The principle underlying this methodology is that the fatigue crack growth data (N, a) occur in a form of a "time" series where observations are dependent [14,15]. This dependency is not necessarily limited to one step (Markov assumption) but it can extend to many steps in the past of the series. Thus, in general, the current value N_a (= number of cycles at crack size a) of the process N can be expressed as a finite linear aggregate of previous values of the process and the present and previous values of a random shock u [16], i.e.

$$N_a = \phi_1 N_{a-1} + \phi_2 N_{a-2} + \cdots + \phi_p N_{a-p} + u_a - \theta_1 u_{a-1} - \theta_2 u_{a-2} - \cdots - \theta_q u_{a-q} \tag{1}$$

In this equation N_a, N_{a-1}, N_{a-2},... and u_a, u_{a-1}, u_{a-2},... represent respectively the number of cycles and the value of the random shock at the indexing equally spaced crack sizes a, $a-1$, $a-2$,.... The random shock u constitute a white noise stochastic process, whose distribution is assumed to be Gaussian with zero mean and standard deviation σ_u.

Defining the autoregressive operator of order p by

$$\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p \tag{2}$$

and the moving-average operator of order q by

$$\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \cdots - \theta_q B^q \tag{3}$$

eqn. (1) can be economically rewritten as

$$\phi(B) N_a = \theta(B) u_a \tag{4}$$

It is recalled that B stands for the backward shift operator defined as $B^s N_a = N_{a-s}$. Another closely related operator, to be used below, is the backward difference operator ∇ defined as $\nabla N_a = N_a - N_{a-1}$ and thus equal to $1 - B$.

In an attempt to physically interpret eqn. (1) or eqn. (4) and connect them to the observed inhomogeneous crack propagation properties, one could associate the autoregressive terms to the mean behavior of each individual test curve and the moving-average terms to the non smoothness within it [13], which is due to the inhomogeneity of the material ahead of the crack tip. In this manner, this spatial irregularity is approximated by the homogeneous random field u [17].

The autoregressive moving-average model (ARMA) as formulated above is limited to modeling phenomena exhibiting stationarity, i.e., broadly speaking, fluctuating about a fixed mean. Clearly, this is not the case for the fatigue crack growth curves of Fig. 2 for which nonstationary processes will have to be employed. It is possible though that, even under these circumstances, the processes still possess a homogeneity of some kind. It is usually the case that the dth difference of the original time series (or, a nonlinear transformation of it) exhibits stationary characteristics. The previous ARMA model could than be applied to the new stationary process $\nabla^d N$ and eqn. (4) will correspondingly read

$$\phi(B) \nabla^d N_a = \theta(B) u_a \tag{5}$$

This equation represents the general model used in this study. Clearly, it can describe stationary ($d = 0$) or nonstationary ($d \neq 0$), purely autoregressive ($q = 0$) or purely moving-average ($p = 0$) processes. It is called autoregressive integrated moving-average (ARIMA) process of order (p, d, q). It employs $p + q + 1$ unknown parameters $\phi_1, \ldots, \phi_p$; $\theta_1, \ldots, \theta_q$; σ_u, which will have to be estimated from the data.

Expecting that the fatigue crack growth curves would eventually reveal some stationary characteristics, the task of estimating the aforementioned unknown parameters is undertaken below. A phenomenological theoretical model will thus be built identifying the mechanism of crack propagation under certain loading and geometrical conditions. An outcome of direct practical importance will evidently be the possibility of forecasting the future behavior of the series N_a from its current and past values. This, of course, will be expressed in a probabilistic manner, in the form of a distribution.

Elaborating briefly on the terminology [16], if the values of N are known up to a current crack size a and a prediction of N is desired for l steps ahead (i.e., at crack size $a + l$), then one refers to "origin a", "lead time l" and "forecasted value $N_a(l)$". The methodology employed is capable of providing, beyond a "best" value of the forecast, probability limits on either side of it for a set of convenient values, for example 50%, 95%. If $a + l$ is chosen to represent a critical value of the crack size, these forecasted results will obviously yield the distribution of the time-to-failure.

The general scheme for determining a model includes three phases, which are:

- model identification, where the values of the parameters p, d, q are defined

- parameter estimation, where the $\{\phi\}$ and $\{\theta\}$ parameters are determined in some optimal way, and
- diagnostic checking for controlling the model's performance.

As is stated however in Ref. [16], there is no uniqueness in the ARIMA models for a particular physical problem. In the selection procedure, among potentially good candidates one is aided by certain additional criteria. Among them are the Akaike's Information Criterion (AIC) and the Schwartz's Bayesian Criterion (SBC) [18]. If $L = L(\phi_1, \ldots, \phi_p, \theta_1, \ldots, \theta_q, \sigma_u)$ represents the likelihood function, formed during the parameter estimation, the AIC and SBC are expressed, respectively, as

$$\text{AIC} = -2 \ln L + 2k \tag{6}$$

$$\text{SBC} = -2 \ln L + \ln (n)k \tag{7}$$

where k = number of free parameters $(= p + q)$ and n = number of residuals that can be computed for the time series. Proper choice of p and q calls for a minimization of the AIC and SBC. Last, in the overall efficiency of the model, the principle of parsimony should be observed. Inclusion of an excessive number of parameters might give rise to numerical difficulties (ill-conditioning of matrices etc.), and might render the model too stiff and impractical.

All numerics of the present work has been carried out using the relevant tools of the SAS package [18], which have proven powerful and reliable. Therefore, no explicit reference is made to the utilized mathematical methods and techniques.

3. MODEL BUILDING—SINGLE SAMPLE FUNCTION

In the search of the parameters p, d, q of the ARIMA model of eqn. (5) we work first with individual crack curves from Fig. 2. For reasons of representativeness, but otherwise randomly, the first curve is chosen terminating with approximately 240000 cycles and lying close to the middle of the plume of the sample functions. This curve is differenced three times. The original curve and its differences are shown in Fig. 3. It is observed that while the second difference starts exhibiting some stationarity, the third produces a quasi stationary signal. Its mean is definitely fixed and equal to zero but its intensity still varies over its length. Similar are also the results of other curves, which are examined as an additional check. The parameter d might thus have the value three.

For confirming this, the autocorrelation functions of the curves of Fig. 3 (record 1) are calculated and shown in Fig. 4. With approximately $m = 89$ observation points it is adequate to plot the autocorrelations using $(m/4) \approx 22$ lags. It is seen that the autocorrelation function of the original crack curve N and its first difference ∇N show pronounced their nonstationary characteristics. They do not die out fast enough and they decay almost linearly. The autocorrelation of $\nabla^2 N$ and even more that of $\nabla^3 N$ die out in an exponential manner. Further, these autocorrelations (and their partial autocorrelations, not shown here) seem to be mixtures of exponentials and damped sine waves. This implies [16] that the sought process must be of the mixed type (containing both autoregressive and moving-average components). It is thus confirmed that $d = 3$ can be used.

For the determination of the parameters p and q it has been deemed appropriate to resort to the aid of AIC and SBC, while for practical reasons it would be desirable to keep q as small as

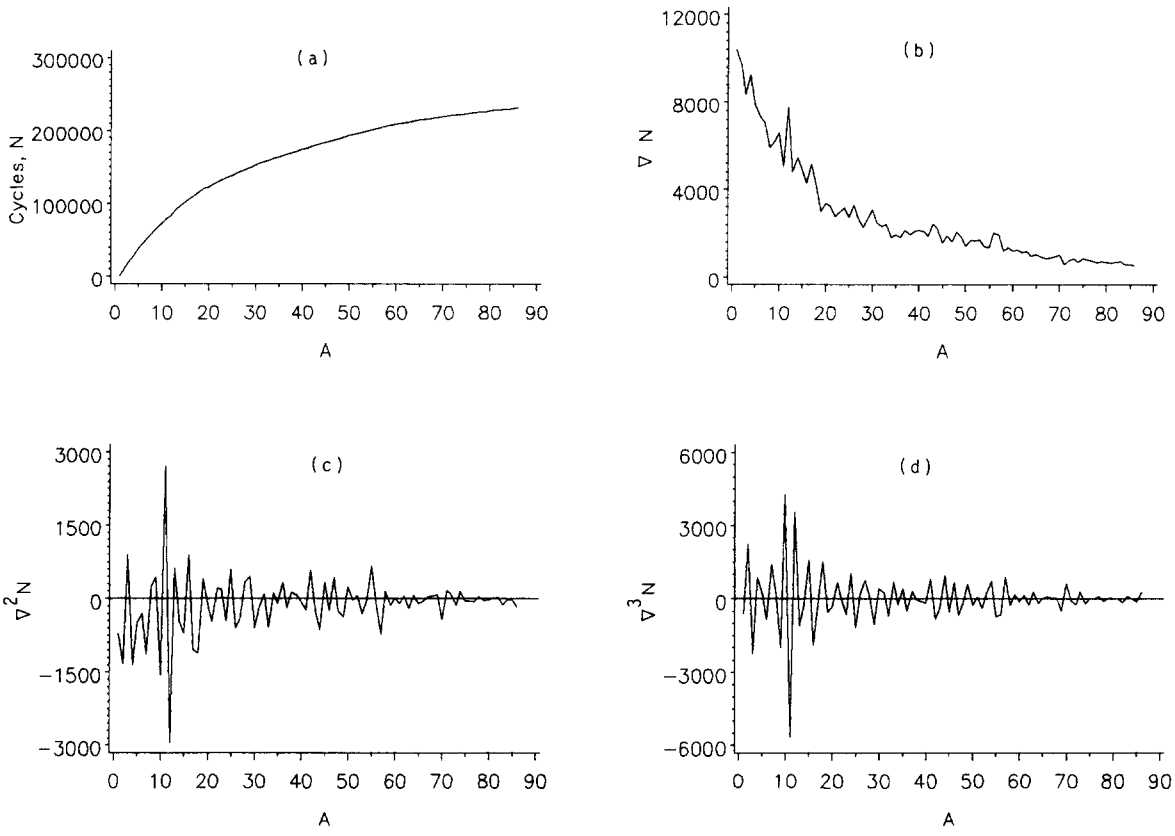

Fig. 3. Original curve and successive differences of record 1.

possible. Thus several models have been tested and their AIC and SBC indices computed with $d = 3$ and ranges of p and q, respectively, $p = 0$–12 and $q = 0$–1. For record 1, these results are plotted in Fig. 5. It is seen that, according to both AIC and SBC, minima appear for $q = 0$ at $p = 5$ and for $q = 1$ at $p = 2$. This latter is actually absolutely smaller. The values of the corresponding parameters are given in Table 1.

The parameter selection is postponed for a while until these two ARIMA models get tested for their predictive capabilities performance. Referring to the previously introduced terminology, this test consists of forecasting the number of cycles N departing from three different origins A, which are chosen to be $A = 40$ ($a = 24.8$ mm), $A = 55$ ($a = 30.8$ mm) and $A = 75$ ($a = 38.8$ mm). The lead times are, correspondingly, selected as $l = 60$, $l = 45$ and $l = 25$, thus producing forecasts up to $a = 48.6$ mm. Clearly, this value lies beyond the utilized data base of Fig. 2, where $a_f = 44.2$ mm, but within the original experimental data, where $a_{max} = 49.8$ mm.

Figures 6 and 7 show these forecasting results for the above two entertained ARIMA models. Three curves are plotted in each graph: the actual data and the forecasted ones referring to the vertical axis on the right, and the residual error (= actual − forecasted value) referring to the vertical axis on the left. The middle broken vertical line indicates the origin. It is very encouraging to see that the residuals are in general small. As expected, the greatest discrepancy appears when one predicts from the early origin of $A = 40$. Even in this case however, the

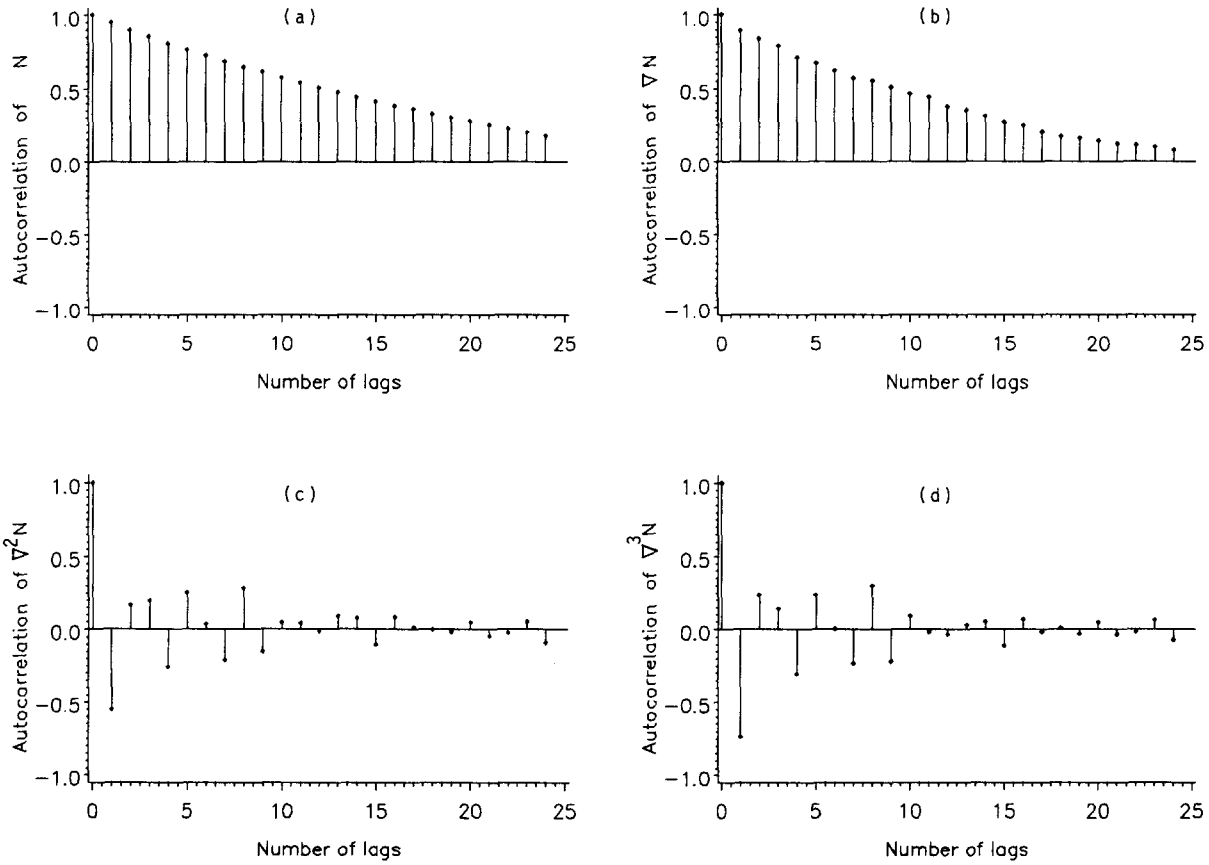

Fig. 4. Autocorrelation function of record 1 and of its successive differences.

situation for shorter forecasts is still good. In addition, the actual curve always falls within the 95% Gaussian probability limit curves which can be drawn on each side of the forecasted one (and not shown here for retaining the figures' legibility).

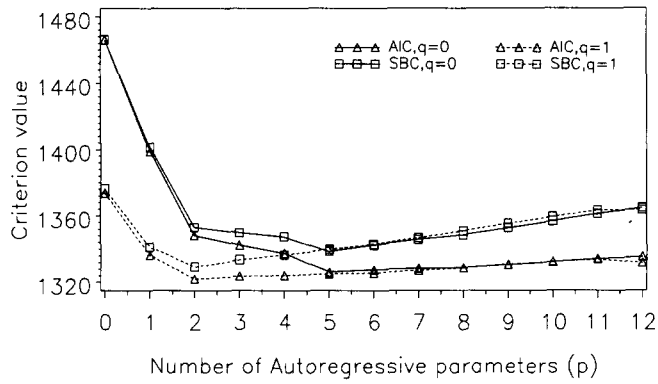

Fig. 5. Behavior of the value of the criteria AIC and SBC for record 1.

Forecasting from the later origins produces, of course, excellent results. In particular, for $p = 2$, $q = 1$ the actual and the forecasted curves are practically indistinguishable. This fact, along with the general philosophy of model building outlined above, leads to the conclusion that the selection $p = 2$, $q = 1$ is the most suitable for the case under consideration.

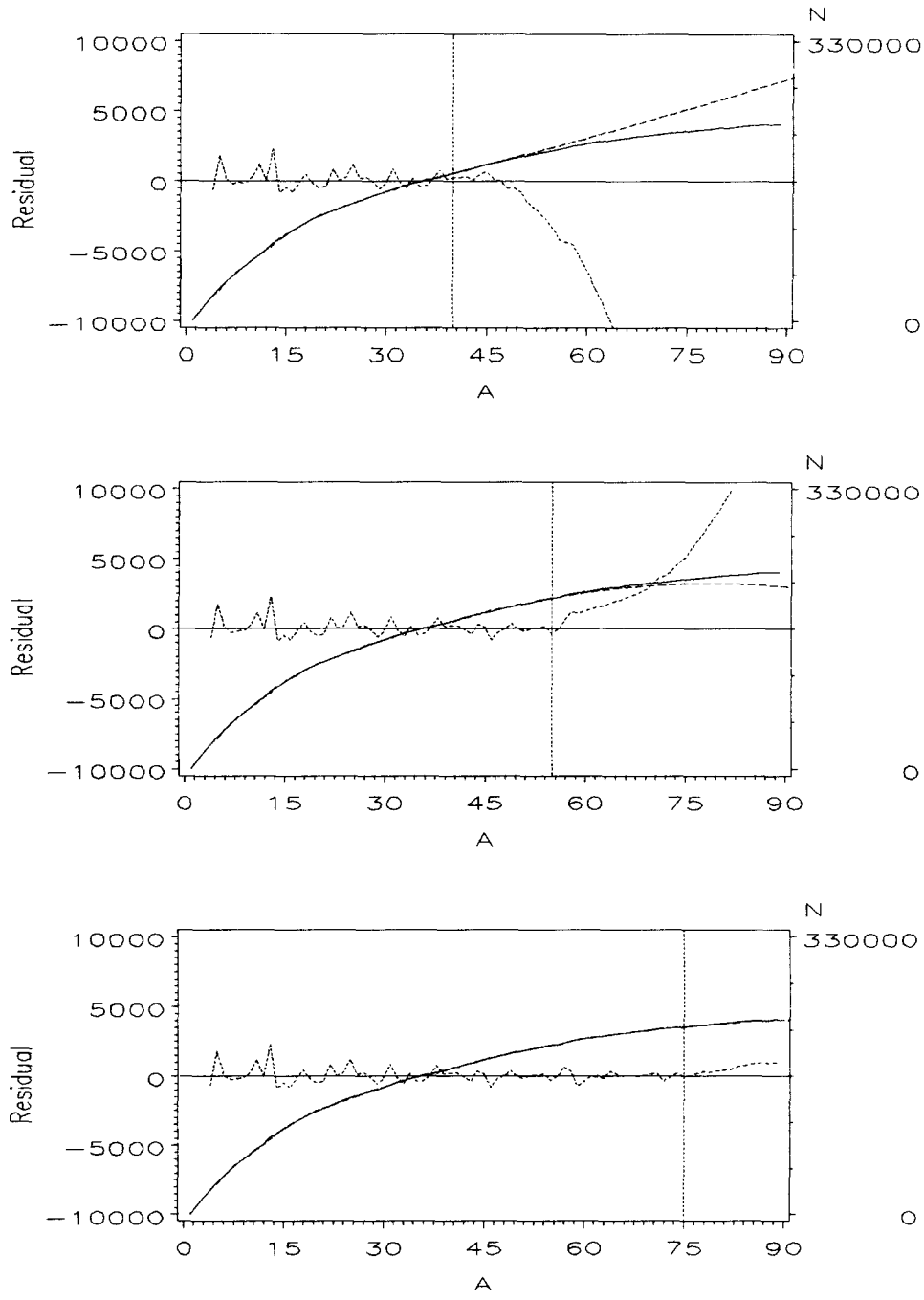

Fig. 6. Forecasting results of record 1, starting from three different origins and using the model $(p, d, q) = (5, 3, 0)$.

Therefore, record 1 can be best reproduced by the process defined by the (2, 3, 1) ARIMA model

$$\left(1-\phi_1 B-\phi_2 B^2\right)\nabla^3 N_a=(1-\theta_1 B)u_a \tag{8}$$

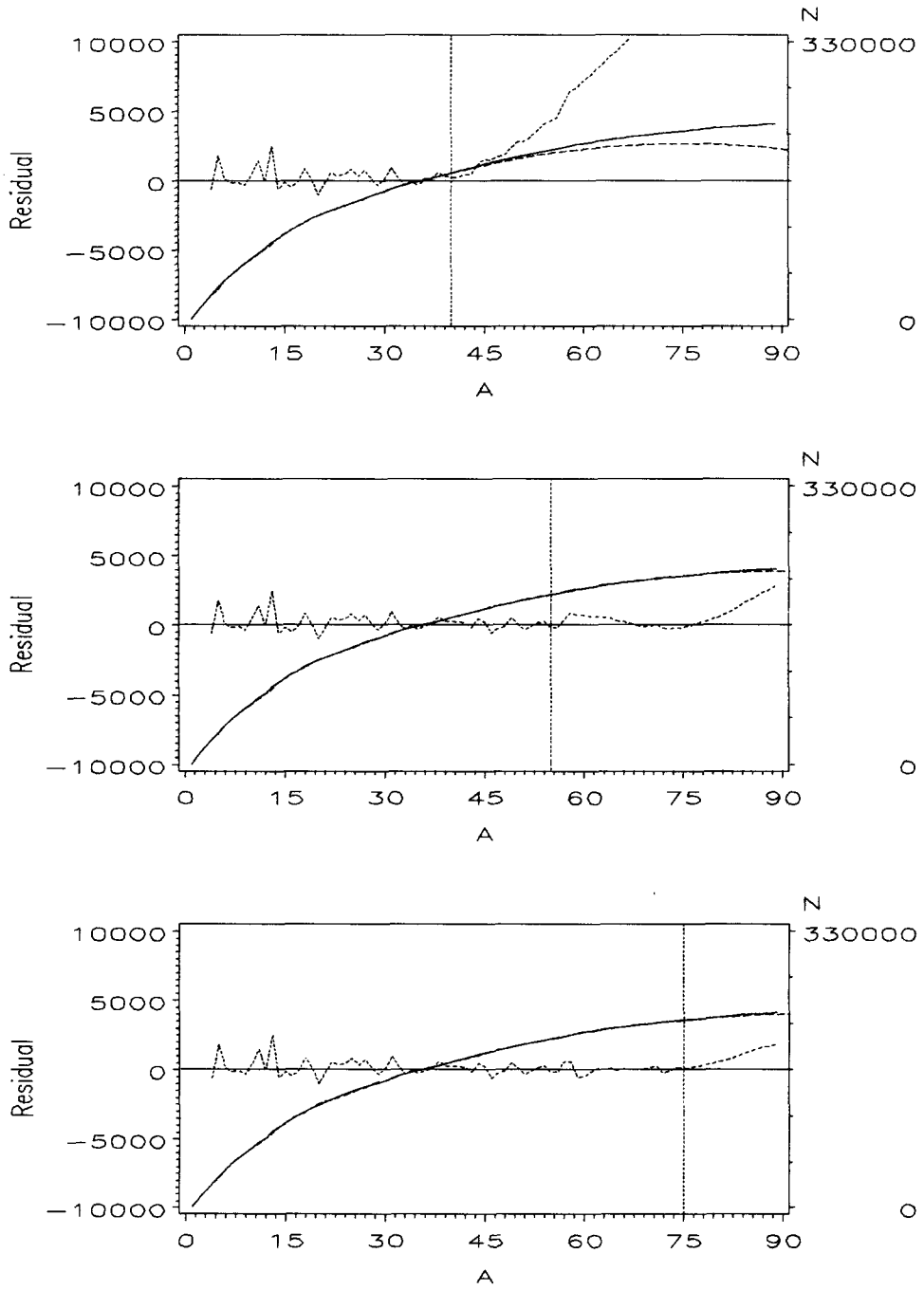

Fig. 7. Forecasting results of record 1, starting from three different origins and using the model (p, d, q) = (2, 3, 1).

TABLE 1

Parameter values of two ARIMA models for record 1

	ARIMA (5, 3, 0)		ARIMA (2, 3, 1)	
	ϕ	θ	ϕ	θ
1	−1.64191	–	−0.92598	0.85170
2	−1.61439	–	−0.44547	–
3	−1.20966	–	–	–
4	−0.85981	–	–	–
5	−0.37683	–	–	–
σ_u	514.6		506.4	
AIC	1326.24		1321.87	
SBC	1338.51		1329.24	

where the parameters take on the relevant values of Table 1. In order to gain more insight into the nature of the model it is instructive to expand the compact form operators of eqn. (8) and rewrite it in full:

$$N_a = (3 + \phi_1)N_{a-1} + (-3 - 3\phi_1 + \phi_2)N_{a-2} + (1 + 3\phi_1 - 3\phi_2)N_{a-3} + (-\phi_1 + 3\phi_2)N_{a-4} - \phi_2 N_{a-5} + u_a - \theta_1 u_{a-1} \quad (9)$$

Upon substitution of the numerical values of the constants for record 1, eqn. (9) yields:

$$N_a = 2.07402N_{a-1} - 0.66753N_{a-2} - 0.44153N_{a-3} - 0.41043N_{a-4} + 0.44547N_{a-5} + u_a - 0.85170u_{a-1} \quad (10)$$

This equation shows that the current N_a can be expressed as a linear combination of its previous five values and of the random deviates of the present and the previous step. The relative weight of each contributor seems to diminish when moving further into the past.

4. MODEL BUILDING—SAMPLE FUNCTION SET

It is evident that equations similar to eqns. (8)–(10) can be obtained for every single record of Fig. 2. At this point one could construct histograms of the parameters ϕ_1, ϕ_2, θ_1 and claim that in this manner the whole set of experimental curves (68) can be represented by the ARIMA (2, 3, 1) model, where the coefficients will be random variables [19]. While this approach yields good results, it is demonstrated below that the so-called seasonal or multiplicative time series can offer a unique tool for data reduction, with particular bearing to the real time monitoring and prediction of crack propagation.

The plume-like appearance of the curves of Fig. 2 can be modified, and they can be seen as one long series consisting of 68 successive curves on the A-axis, as shown in Fig. 8a. Clearly, now $1 \leqslant A \leqslant 89$ refers to record 1; $90 \leqslant A \leqslant 178$ refers to record 2; and so on. Under this perspective, it is amply revealed that similarities occur every 89 crack intervals and thus this series exhibits periodic behavior with period $s = 89$. In this type of series one expects relationships to exist (i) between observations of successive number of cycles in a particular record (the previously tackled problem); and (ii) between observations for the same crack size in successive records.

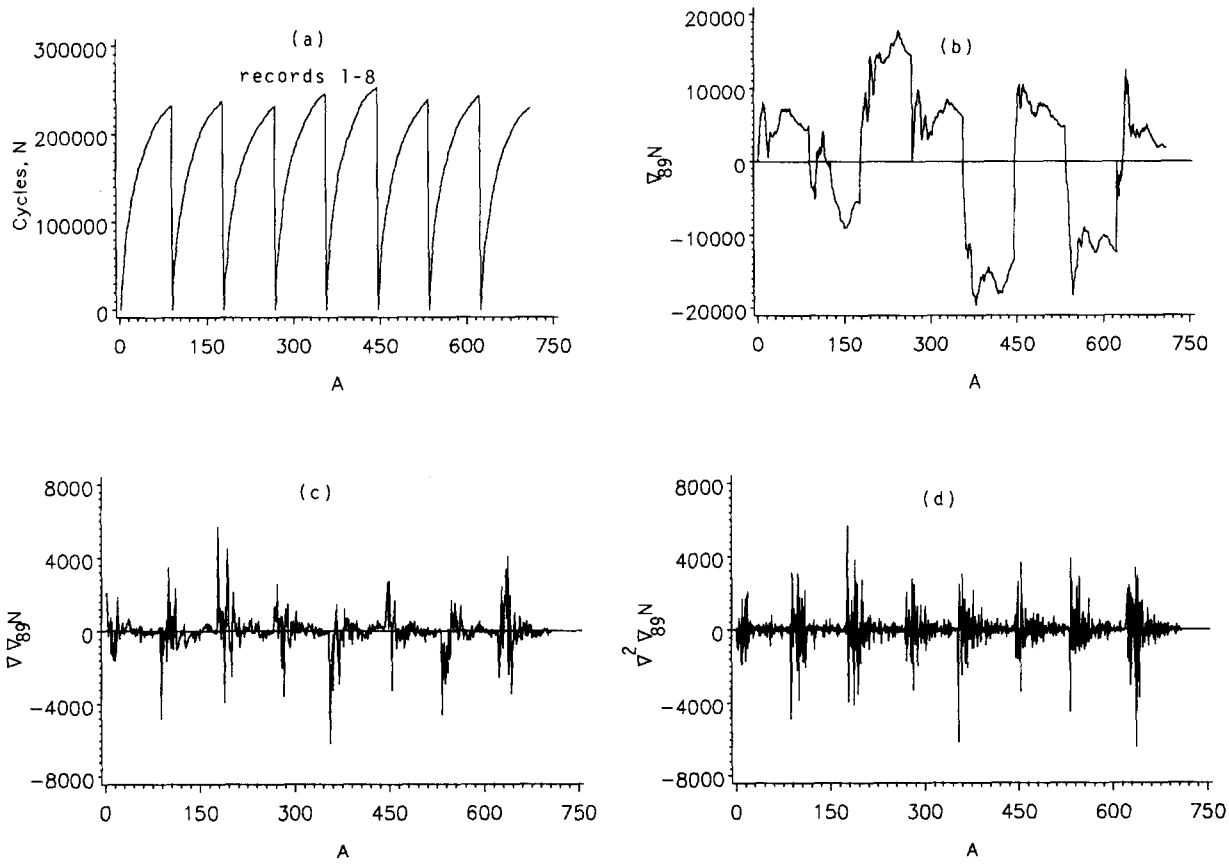

Fig. 8. (a) Sequential presentation of some crack curves; and their successive differences with respect to (b) record; (c) crack size; and (d) crack size.

Starting from the ARIMA model of eqn. (5), it can be deduced [16] that a seasonal series can be mathematically represented by the general multiplicative model

$$\phi_p(B)\Phi_P(B^s)\,\nabla^d\,\nabla_s^D N_A = \theta_q(B)\Theta_Q(B^s)u_A \tag{11}$$

In this equation the parameters p, d, q and the operators $\phi_p(B)$ and $\theta_q(B)$ are exactly as those defined for eqn. (5) and refer to the afore-discussed point (i), while $\nabla_s = 1 - B^s$. $\Phi_P(B^s)$ and $\Theta_Q(B^s)$ are proper polynomials in B^s of degrees P and Q, respectively, representing relationships of point (ii) above. This multiplicative process is said to be of order $(p, d, q) \times (P, D, Q)_s$.

The building of the model for the specific physical problem follows again the same steps: identification, estimation, diagnostic checking. Guided by the experience gained when working with a single record, it is expected that the values of d and D should be searched, respectively, in the intervals [1–3] and [0–1]. Indeed, as seen in Fig. 8d, when $d = 2$ and $D = 1$ a nice stationary looking signal is retrieved. It is clarified that this last plot results after three differences: the first with respect to record ($D \times s = 1 \times 89 = 89$) produces $\nabla_{89}N_A$ (Fig. 8b); the second with respect to crack size produces $\nabla\nabla_{89}N_A$ (Fig. 8c); the third with respect to crack size again produces $\nabla^2\nabla_{89}N_A$ (Fig. 8d). The stationarity characteristics of this signal are definitely superior to those of the single record, Fig. 3d. This fact is, of course, due to the signal's new length after having

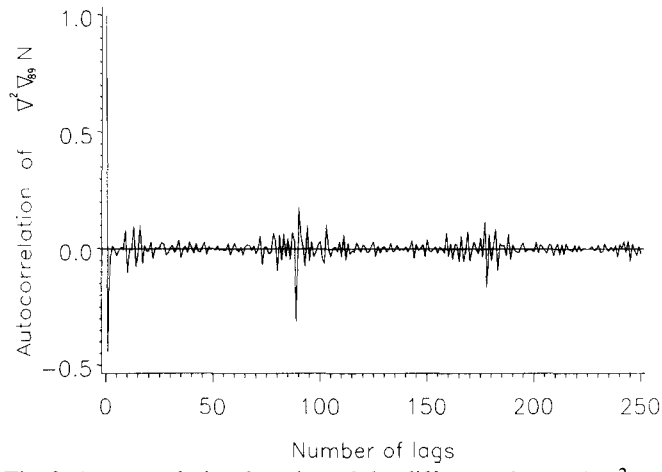

Fig. 9. Autocorrelation function of the differenced record $\nabla^2\nabla_{89}N_A$.

changed our viewpoint. It is expected that this matter will have a favorable effect on the order of the seasonal model to be selected.

The autocorrelation function of $\nabla^2\nabla_{89}N_A$, shown in Fig. 9, exhibits all proper characteristics: it initially dies out exponentially, shows a substantial correlation at lag 89, dies out anew, shows a smaller correlation at lag 178, and so on. In the process of the parameter estimation, values smaller than before for the p and q are tried out, and P and Q are not expected to be greater than unity. $P = 0$ or $P = 1$ means (when $D = 1$) that the seasonal model of eqn. (11) will include information from the previous record or, from the two previous records, respectively.

In fact, running several tests, it is seen that the model with $p = 1$, $q = 1$, $P = 1$, $Q = 1$ shows good forecasting capabilities. Having used the first four records, an excellent forecast is obtained to the end of the fourth record even when departing from an early origin within it, Fig. 10a. Equally satisfactory is the performance of the more parsimonious model with $p = 1$, $q = 1$, $P = 0$, $Q = 1$, Fig. 10b. These latter parameter values are also checked and found to satisfy the requirements of the other supplementary criteria (AIC, SBC). On these grounds it is concluded that the multiplicative model of eqn. (11) of order $(1, 2, 1) \times (0, 1, 1)_{89}$ can efficiently represent the physical phenomenon of crack propagation as represented by the curves of Fig. 2.

The analytical expression of the proposed model is explicitly

$$(1 - \phi_1 B)\,\nabla^2\,\nabla_{89}N_A = (1 - \theta_1 B)\left(1 - \Theta_1 B^{89}\right)u_A \tag{12}$$

or, in its expanded form

$$\begin{aligned} N_A = {} & (2 + \phi_1)N_{A-1} - (1 + 2\phi_1)N_{A-2} + \phi_1 N_{A-3} \\ & + N_{A-89} - (2 + \phi_1)N_{A-90} + (1 + 2\phi_1)N_{A-91} - \phi_1 N_{A-92} \\ & + u_A - \theta_1 u_{A-1} - \Theta_1 u_{A-89} + \theta_1\Theta_1 u_{A-90} \end{aligned} \tag{13}$$

Equation (13) indicates which are the contributors to the formation of the current value of N_A. There clearly participate the current record (with 3 terms), the previous record (with 4 terms), the current noise (with 2 terms) and the noise from the previous record (with 2 terms). As a general rule, it is expected again that the relative weight of each contributor will decrease as one moves into the past of a record and into the preceding one.

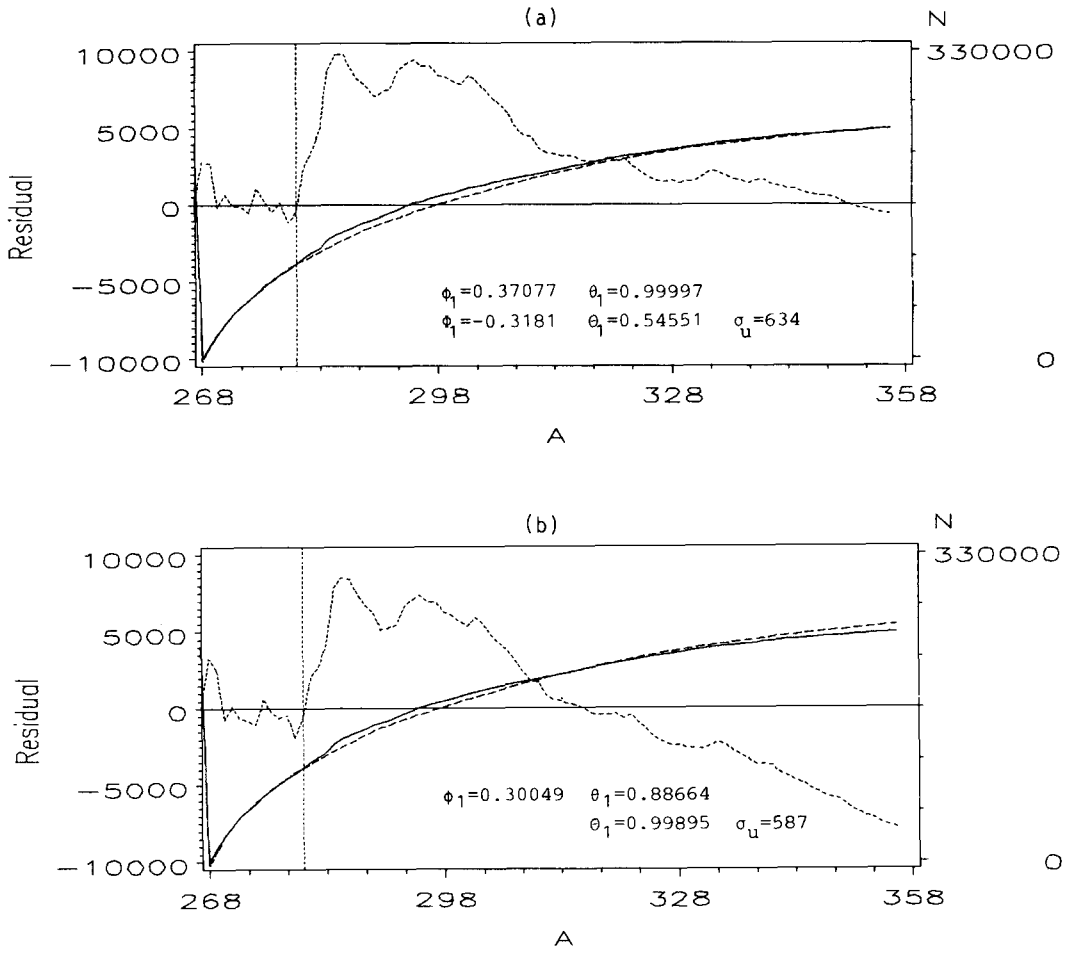

Fig. 10. Forecasting of the fourth record, using the multiplicative model $(1, 2, 1)\times(1, 1, 1)_{89}$, and $(1, 2, 1)\times(0, 1, 1)_{89}$, respectively.

Equation (13), as it stands above, can reproduce effectively the sequence of crack histories, which have been used for its parameter estimation. However, it must be noted that when forecasting a crack behavior, it is not expected on physical grounds that the values (N, a) of a new specimen would depend in some preferential manner on the values (N, a) of the immediately previous tested specimen. On the contrary, the order of considering the specimens should play no role at all. For this reason, it seems a lot more rational that the "previous" record in eqn. (13) be replaced by the "average curve" of all available records. Furthermore, the parameter values should also result independent of the records' order. For the data set under consideration, Fig. 11 shows how these parameters vary with respect to the number of records considered. It is observed that as the number of records increases, the parameters converge to some limiting values. For all 68 curves, these are $\phi_1 = 0.08544$, $\theta_1 = 0.99999 \approx 1$ and $\Theta_1 = 0.65878$, while $\sigma_u = 1861$.

Illustrating these last points and the new interpretation of eqn. (13), one more test is carried out demonstrating the performance of the model when monitoring and forecasting the outcome of an experiment. Suppose, for example, that a new specimen is tested and that its crack history

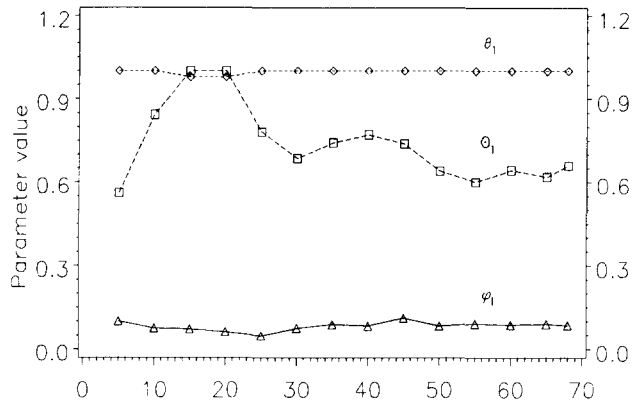

Fig. 11. Convergence of the multiplicative model $(1, 2, 1)\times(0, 1, 1)_{89}$ parameters for the 68 records.

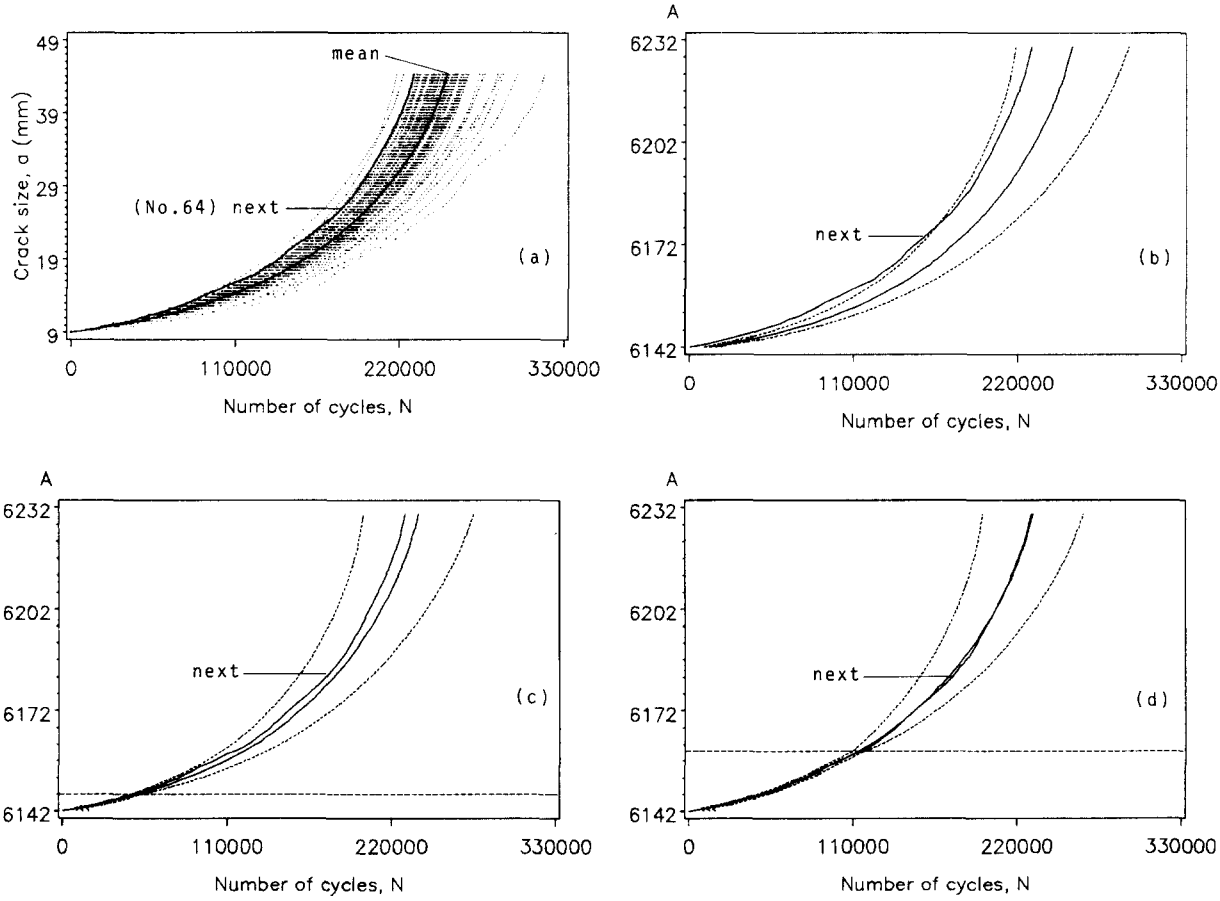

Fig. 12. (a) Available curves, their mean and the next experiment (record 64); (b) its a priori forecasting; (c) its forecasting when 5% of the measurements is included; and (d) its forecasting when 20% of the measurements is included.

curve, denoted as "next", is as shown in Fig. 12a, which is chosen to be identical to that of specimen 64 of Fig. 1. Clearly, this curve is situated away from the average curve (denoted as "mean"), and it represents a rather extreme case. It is intended to predict this crack's behavior via the model of eqn. (12) as established above for the 68 curves of Fig. 2.

Fig. 12b shows an a priori forecasting of the full curve (i.e., origin at $A = 6142$, corresponding to $a = 9$ mm, and lead equal to $l = 89$), along with the 95% Gaussian probability limits on each side. Naturally, this prediction falls very close to the average curve of Fig. 1. Figures 12c and 12d show the same forecasting, where some of the actual measurements (N, a) for the experiment have gradually been incorporated (5%, 20% of the measurements, respectively). It is seen that as more and more actual data keep coming in, the forecasted curve moves closer to the real one. The 95% probability interval also becomes narrower. In conclusion, an initially remote forecast can be continuously updated and, as the lead shortens, greater accuracy can be achieved. This matter can have important consequences for real components or structures containing cracks operating under similar conditions to specimens for which a model like that of eqn. (12) is available.

Thus we see that the simple model of eqn. (12), containing only three parameters, reproduces and forecasts the crack growth curves surprisingly well. Its potentiality is due to its capability to assimilate efficiently the pattern of evolution of the physical process from the previous records, and to utilize effectively any new acquired information. For the model to be operative, the knowledge of at least 92 values of N (that is, the average record plus 3 points of the new test) is required. It is recalled that the model of eqn. (8) for the single record required only the knowledge of the five previous values of N.

5. CONCLUSIONS

Autoregressive integrated moving-average processes have been employed for the modeling of the number of cycles over the crack size for the fatigue crack propagation phenomenon. Even though no perfect stationarity conditions have been obtained in the treatment of the Virkler records, an overall good preformance of the derived models has been observed. It has been found that a single record can be reproduced satisfactorily by an ARIMA process of order $(p, d, q) = (2, 3, 1)$. The quality of the forecasts depends upon the origin; an early origin allows for short forecasts while a later origin yields unconditionally good forecasts. A multiplicative ARIMA process of order $(p, d, q) \times (P, D, Q)_s = (1, 2, 1) \times (0, 1, 1)_{89}$ has been found to represent very efficiently the whole set of the fatigue crack records. Its forecasting capabilities are excellent both at reproducing existing data, and at the monitoring and prediction of new experiments.

It can therefore be claimed that quite efficient models for the fatigue crack growth phenomenon have been constructed. In addition, they have the advantage of being compact, easily presentable and implementable. They can thus serve in practical situations, as they can readily furnish updated predictions of a component's residual lifetime after periodic inspections.

Every such model is built based on the primary form of information of the crack growth, i.e. the (N, a) sample functions, and consequently is suitable for a specific set of geometric and loading conditions. The possibility of utilizing the same model under different conditions, or of attaching physical significance to its parameters, remains, of course, to be investigated. Towards this direction, the approach suggested by the authors of Refs. [14,20] for their phenomenological FCG model can be worth following. Finally, it is noted that the performance of the models may

be improved (at the expense of their simplicity) by introducing some nonlinear transformation of the data [21], which would produce closer to stationarity conditions after differencing, or by employing schemes of ARMA algorithms [22–24] especially devised for nonstationary processes.

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