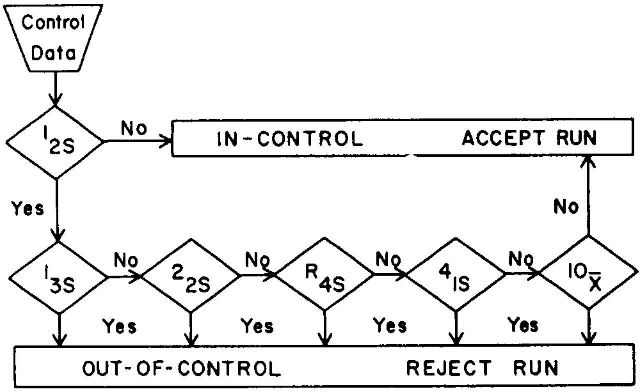

Fig. 1. Logic diagram for applying a series of decision criteria (control rules) in the multi-rule Shewhart procedure

run is out-of-control. The particular rule violated may give some indication of the type of analytical error occurring. Random error will most often be detected by the 1_{3s} and R_{4s} rules. Systematic error will usually be detected by the 2_{2s}, 4_{1s}, or $10_{\bar{x}}$ rules and, when very large, by the 1_{3s} rule.

Calculation of Control Limits

The mean ($\bar{x}$) and standard deviation (s) are calculated from the following equations:

$$\bar{x} = \Sigma x_i / n \tag{1}$$

$$s = \sqrt{\frac{n\Sigma x_i^2 - (\Sigma x_i)^2}{n(n-1)}} \tag{2}$$

where x_i is an individual control observation and n is the total number of control observations collected in the time period being analyzed. Initial estimates are often made from a data set where n is approximately 20. When n is this low, these estimates may not be reliable. They should be revised when more control observations are accumulated. This can be done by analyzing additional data sets and recording n, Σx_i, and Σx_i^2. The cumulative totals for these terms can be obtained by adding the values for the different data sets. Then these totals can be used in equations 1 and 2 to give cumulative estimates of $\bar{x}$ and s.

Table 1. Example Control Observations for One Control Material during Five One-Month Periods

Day	Month 1	2	3	4	5
1	98	100	97	101	100
2	97	109	98	100	96
3	95	102	102	99	101
4	103	104	92	100	102
5	100	97	104	96	104
6	104	105	100	100	100
7	92	98	95	98	96
8	94	100	100	97	101
9	102	96	104	103	99
10	95	103	101	107	105
11	100	97	101	104	100
12	93	97	99	96	95
13	100	96	97	104	101
14	106	97	112	105	99
15	112	104	92	101	90
16	94	99	105	102	98
17	96	105	105	102	106
18	97	94	101	102	100
19	103	95	95	101	101
20	104	97	100	104	97

Table 2. Monthly and Cumulative Means and Standard Deviations Calculated from Control Data in Table 1

	Monthly (and cumulative) totals			Calculated statistics	
Month	n	x_i	x_i^2	$\bar{x}$	s
1	20	1985	197 507	99.25	5.11
2	20	1995	199 319	99.75	4.09
	(40)	(3980)	(396 825)	(99.50)	(4.46)
3	20	2000	200 434	100.00	4.78
	(60)	(5980)	(597 259)	(99.67)	(4.61)
4	20	2022	204 592	101.10	2.97
	(80)	(9002)	(801 851)	(100.00)	(4.29)
5	20	1991	198 457	99.55	3.65
	(100)	(9993)	(1 000 308)	(99.93)	(4.15)

Control limits are calculated from $\bar{x}$ and s as follows:

$$3s \text{ control limits} = \bar{x} \pm 3s \tag{3}$$

$$2s \text{ control limits} = \bar{x} \pm 2s \tag{4}$$

$$1s \text{ control limits} = \bar{x} \pm 1s \tag{5}$$

The calculation of control limits is illustrated by the data in T^bles 1–3. Table 1 shows control data collected during five months, 20 observations per month. Table 2 shows the calculated means and standard deviations, both for the individual monthly data sets and for the accumulated data. For example, for month 2, the first line gives n, Σx_i, and Σx_i^2, and the corresponding mean and standard deviation calculated from those values. The next line gives the cumulative values and the corresponding mean and standard deviation. The cumulative values for n, Σx_i, and Σx_i^2 are obtained by adding the values in the previous two lines. Observe that the standard deviation changes more from month to month for the individual monthly data sets than for the cumulative data. For these simulated data, the true mean was specified to be 100 and the true standard deviation 4.00. Note that the accuracy of the estimates improves as the cumulative number of observations increases. This shows that the control limits that are calculated from the cumulative values will be more reliable than those calculated from the individual monthly data sets. In Table 3 the calculated control limits are compared.

Preparation of Control Charts

The y-axis should be scaled to provide a concentration

Table 3. Control Limits Calculated for the Control Data in Table 1, with Use of the Means and Standard Deviations from Table 2

	Monthly (and cumulative) control limits		
Month	$\bar{x} \pm 1s$	$\bar{x} \pm 2s$	$\bar{x} \pm 3s$
1	94.1–104.4	89.0–109.5	83.9–114.6
2	94.7–103.8	91.6–107.9	87.5–112.0
	(95.0–104.0)	(90.6–108.4)	(86.1–112.9)
3	95.2–104.8	90.4–109.6	85.7–114.3
	(95.0–104.3)	(90.4–108.9)	(85.8–113.5)
4	98.1–104.1	95.2–107.0	92.2–110.0
	(95.7–104.3)	(91.4–108.6)	(87.1–112.9)
5	95.9–103.2	92.3–106.8	88.6–110.5
	(95.8–104.1)	(91.6–108.2)	(87.5–112.4)