The DeProF Theory for Two-Phase Flow in Porous Media: Conceptual Framework and Experimental Validation

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Abstract *The Decomposition into Prototype Flows (DeProF) theory provides a mechanistic and thermodynamically consistent framework for describing steady-state immiscible two-phase flow in porous media. Developed by M.S. Valavanides and collaborators over two decades, DeProF replaces empirical relative permeability models with a physically based, statistical-thermodynamic description linking pore-scale flow configurations to macroscopic transport laws. DeProF treats steady-state flow as a statistical ensemble of prototype microflows, defines configurational entropy and system invariants, and predicts flow-rate-dependent scaling of relative permeabilities. Recent laboratory and microfluidic experiments confirm these predictions, establishing DeProF as a universal framework for understanding and scaling two-phase flow behavior across flow regimes for any two-fluid and pore network system.*

1. Introduction

Traditional continuum approaches on modeling of immiscible two-phase flow in porous media rely on rate-independent relative permeability functions derived empirically, offering limited predictive power. The *DeProF* (*Decomposition into Prototype Flows*) theory (Valavanides & Payatakes 2001–2004) was introduced to overcome these limitations by explicitly linking the *microscopic flow topology* with *macroscopic transport properties***,** namely, capillary number, Ca, and flowrate ratio, r, through a *true-to-mechanism* representation. The theory has since evolved into a statistical-thermodynamic model, linking basic pore-scale mechanisms to macroscale observations, revealing universal flow behavior and system invariants, aspects of energy efficiency and also inferring a universal scaling law, established on flowrate dependency (Valavanides 2012, 2018, 2023). On-going experimental work (2016–2025) has tested these predictions across model pore-network and core-scale systems.

2. Theoretical Framework of the DeProF Model

2.1 Origin and Conceptual Basis

The DeProF model was first articulated by Valavanides & Payatakes (2001; 2000) as a *true-to-mechanism* model of steady-state two-phase flow. Complex flow fields in a porous medium are decomposed into a finite number of *prototype microflows*— namely connected flow, ganglion dynamics and drop traffic flow, taking into account wetting-film flow— that recur throughout the pore network.

Each prototype flow is characterized by local capillary and viscous interactions. DeProF implements hierarchical modelling from pore to Darcy scales. At the pore scale, unit-cell conductivities are estimated by considering viscous and capillary resistances. Mechanistic analysis includes *dynamic hysteresis* and *flow intermittency*. The former by assigning different dynamic contact angles per the advancing and receding menisci; the latter by imposing criteria of mobilization probability based on local pressure gradients. All possible unit-cell flow configurations are considered and the mixture of the three prototype flows is accounted by respective volumetric fractions, as free variables. Then, upscaling is achieved by implementing effective medium theory and examining pore to Darcy scale consistency between expressions of mass and momentum balances for every combination of volumetric fractions of the constituent prototype flows. The flow configurations that satisfy the aforementioned multi scale consistency-check comprise the ensemble of physically admissible flow configurations. By postulating ergodicity, the macroscopic flow incessantly visits all physically

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admissible configurations within this ensemble with equal probability. Therefore, the global flow emerges as a *statistical ensemble* of these local states.

When the flow settles in *fully-developed state*, the mobility ratio is equal to the flowrate ratio, and the global flow is described by the reduced pressure gradient, x, equivalent to the inverse of relative permeabilities, k_{rw} and k_{rn}, as in terms of the externally imposed macroscopic flow conditions, and the physicochemical properties of the two-fluid and pore network system, namely the viscosity ratio, M, the interfacial tension, γ_{nw}, the dynamic contact angles, θ_A, θ_R, as well as the network topology and geometry, $\boldsymbol{a_{pm}}$.

$$x = 1/k_{rw} = f\left(Ca, r; M, \gamma_{nw}, \theta_A, \theta_R, \boldsymbol{a_{pm}}\right) = Mr/k_{rn} \tag{1}$$

Later developments (Valavanides & Payatakes, 2002; 2004) incorporated effects of wetting-films as well as network dimensionality. In a complementary work, Valavanides & Payatakes (2003) introduced flow analysis in terms of *energy efficiency* and revealed the existence of *optimum operating conditions*, whereby the flowrate of the non-wetting phase per unit of total energy dissipation takes local maximum values. In that context, a latent correlation between energy efficiency and plurality of constituent microflows emerges. Valavanides (2010) inferred a conceptual relation between maximum energy efficiency, thermal entropy (as heat released to the environment) and configurational entropy (as ensemble of interstitial, constituent microflows),

$$S_{UNIV} = S_{SYS} + S_{SUR} = k_{ss2fpm} \ln N_{PAS}(Ca, r) + Q(Ca, r)/T_0 \tag{2}$$

where $S_{UNIV}, S_{SYS}, S_{SUR}$ represent the total entropy of the environment comprising the entropy produced within the biphasic flow and network system, and the entropy produced by heat release, $Q(Ca, r)$, in its surroundings at temperature T_0. Reflecting on the first term, k_{ss2fpm} represents a Boltzmann-type constant pertaining to the biphasic flow process, and $N_{PAS}(Ca, r)$ the number of the corresponding microstates. The latter comprise the ensemble of the, so-called, *physically admissible solutions* in the DeProF equation, expressing the pore-to-macro-scale consistency between the interstitial flow structure and the externally imposed macroscopic flow conditions.

The synthesis by Valavanides (2012), provided an overview of the DeProF evolution and proposed it as a potential bridge between pore-scale mechanics and macroscopic thermodynamics.

2.2 Statistical-Thermodynamic Extension

Building on these mechanistic foundations, Valavanides & Daras (2016) defined and counted the configurational microstates of steady-state two-phase flow, introducing a formal, Boltzmann-Gibbs type expression for configurational entropy to quantify the degree of flow organization.

$$\begin{aligned} S_{SYS}(Ca, r) &= k_{DeProF} \ln P(Ca, r) \\ &= k_{CPF} \sum_{j=1}^{N_{PAS}(Ca,r)} \left[-\beta_j' \ln \beta_j' - \left(1-\beta_j'\right) \ln\left(1-\beta_j'\right)\right] \\ &+ k_{DOF} \sum_{j=1}^{N_{PAS}(Ca,r)} \left(1-\beta_j'\right) \left[-\left(1-\omega_j'\right) \ln\left(1-\omega_j'\right) \right. \\ &\left. - \left(\omega_j' \ln \omega_j' - \omega_j' \left(\sum_{i=1}^{I_{max}} n_i' \ln n_i' \right)_j \right) \right] \end{aligned} \tag{3}$$

where $k_{DeProF}, k_{CPF}, k_{DOF}$ represent tentative Boltzmann-type constants pertaining to the entire flow and the constituent Connected Pathway and Disconnected Oil Flows, β_j', ω_j' the associated

volumetric fractions and n_i' the ganglion size distribution, as predicted by the ensemble of the physically admissible solutions of the DeProF equation in terms of the imposed flow conditions (Ca, r).

This allowed two-phase flow to be described using statistical thermodynamics, suggesting the possibility of correlating the conditions of maximum energy efficiency with those of maximum entropy production, proposing a conceptual analogy to the MEPP.

2.3 Flow-Structure Predictions

Implementing systematic, DeProF-based simulations, extending over 4 orders of magnitude over the independent variables, capillary number, Ca, and flowrate ratio, r, and for various viscosity ratio, M, systems, Valavanides (2018a) generated flow-structure maps that describe how oil fragmentation, interfacial area, and surface transport evolve with increasing Ca and r. These maps explain the mechanistic origin of relative-permeability hysteresis and delineate the progressive transition between distinct flow regimes —connected, intermittent and fragmented— each associated with unique energy-dissipation signatures.

2.4 Universal Energy-Efficiency Map and Independent Variables

In a comprehensive 2018 review (Valavanides 2018b), the DeProF theory was recast considering the phenomenology of the sought process, supplementing it with an analysis based on energy efficiency, around the capillary number, Ca, and flow rate ratio, r, in the form

$$f_{EU}(Ca, r) = rk_{rw}/(r+1) = k_{rn}/[M(r+1)] \tag{4}$$

where $f_{EU}(Ca, r)$ is the energy utilization factor, accounting for the energy efficiency of the process operation at Ca and r.

The detailed framework captures transitions in flow topology and identifies *optimum operational conditions* corresponding to *maximum energy efficiency*. The study's rationalized energy efficiency analysis delivered a *Universal Energy Efficiency and Relative Permeability Map*. The latter correlates macroscopic flow efficiency and relative permeabilities, with dissipation mechanisms and defines *Critical Flow Conditions*, forming a unique locus, $r^*(Ca)$, for every two-fluid and pore network system.

The CFC locus can be used to segregate fragmented from connected flow regimes and/or capillarity-dominated, transient or viscosity-dominated biphasic flows. In that context, it stands as an improved version of Lenormand's classification map of pore-scale flow regimes. Moreover, it was shown that the loci of critical flow conditions for different viscosity ratio systems, collapse into a single, master locus (for $M = 1$), when rescaled to an *effective capillary number*, Ca_R. The latter is consistent to the essential physical definition of the capillary number, incorporating the viscous resistances from both fluids over the actual capillary resistances from wettability and hysteresis. In practice, its value can be recovered by systematic co-injection experiments at different flow conditions (hence the adjective "effective").

2.5 Flowrate Dependence and System Invariants

The most recent contribution (Valavanides, 2018c; 2023) formalized the *flowrate dependency* of steady-state, fully-developed, two-phase flow. Tapping on the extensive DeProF simulations (Valavanides 2018a), a *universal relative-permeability scaling function* was recovered along with *system-characteristic invariants*.

The universal relative-permeability scaling function incorporates the effects of capillarity and viscosity in decoupled form, as

$$x(Ca, r) = 1/k_{rw} = A(\log Ca) + Mr \quad , \qquad k_{rn} = rMk_{rw} \tag{5}$$

where the kernel function , $A(\log Ca)$, namely the *Intrinsic Dynamic Capillary Pressure* (*IDCP*) function, describes exclusively the flowrate dependence of the capillary resistances, Figure **2**(a).

The proposed scaling and, in particular, the structure of the IDCP curve, may provide insight into the interstitial flow structure, Valavanides et al., 2020.

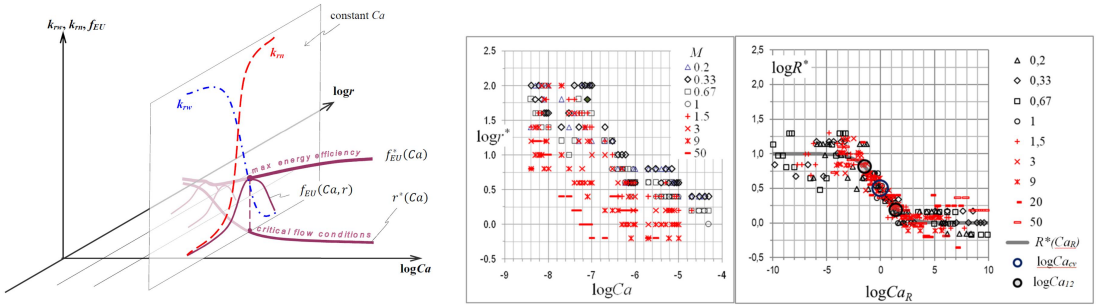
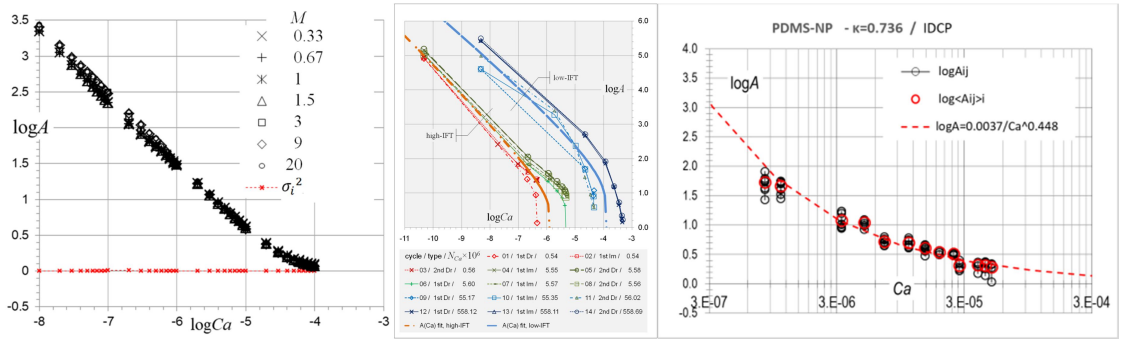


Figure 1: Illustration of the key results of the energy efficiency analysis within the DeProF theory: Left: the reduced energy efficiency, $\boldsymbol{f_{EU}}$, and relative permeabilities, $\boldsymbol{k_{rw}}, \boldsymbol{k_{rn}}$, map in terms of capillary number, $\boldsymbol{Ca}$, and flowrate ratio, $\boldsymbol{r}$. Right: Diagrams of *DeProF* detected critical flow conditions for different viscosity ratio, $\boldsymbol{M}$, systems; the markers pertaining to critical flow conditions, plotted against the conventional capillary number, $\boldsymbol{Ca}$, collapse into a relatively narrow-banded swarm, when plotted against an effective capillary number, $\boldsymbol{Ca_R}$. Source (Valavanides, 2018).



(a) (b) (c)

Figure 2: Intrinsic Dynamic Capillary Pressure (IDCP) curves recovered from SCAL type steady-state, co-injection experiments. (a) virtual experiments implementing the DeProF model algorithm for different viscosity ratio systems (Valavanides, 2023); (b) SCAL experiments, Clasach sandstone core (Valavanides et al., 2020); (c) Right, physical experiments, planar, microfluidic model network (Mouravas et al., 2025).

System-characteristic invariants demonstrate that steady-state, fully developed flow properties are intrinsically rate-dependent yet universally scalable. Among the revealed system-specific invariants are the crossover relperms in terms of Ca, the ceiling of efficiency attained at very high Ca values and the corresponding value of the flowrate ratio, etc. These intrinsic properties were inferred analytically (Valavanides, 2018b), revealed by the extensive DeProF model simulations (Valavanides, 2023) and validated through SCAL type experiments (Valavanides et al., 2016; 2020; Mouravas et al., 2025).

3. Experimental Validation of DeProF Predictions

In the first systematic assessment of the DeProF theory inferences, Valavanides et al. (2016) analyzed 180 steady-state relative-permeability diagrams for various two-phase systems, published in 35 laboratory studies. The observed energy-efficiency envelopes mirrored DeProF predictions: the occurrence of optimum flow conditions and pertinent systematic trends in terms of energy efficiency analysis.

Subsequent SCAL experiments (Valavanides et al., 2020) at IFP Energies Nouvelles verified that relative permeabilities are indeed flow-dependent. The original SCAL data, rescaled according to the DeProF model provisions, exhibited universality, supporting the predicted flow-dependent scaling law, which was at that time already in development phase (Valavanides, 2018c). Moreover, it was shown that analyzing the macroscopic (ex-core) measurements (relative permeability values) by implementing the new, rate-dependent, universal scaling law, it is possible to extract the IDCP curve and reveal the structure of the interstitial flow, independently from sophisticated, invasive imaging technologies, Figure **2**(b).

Using microfluidic pore networks, Karadimitriou et al. (2023) directly visualized flow patterns and measured relative permeabilities across flow rates. The results reproduced DeProF's universal scaling and demonstrated the transition from connected to fragmented flow regimes, validating the universal scaling function proposed by Valavanides (2023). Most recently, Mouravas et al. (2025) extended validation across multiple model networks under SCAL-type protocols. These experiments also confirmed flow-rate dependence in steady-state two-phase flow and verified that DeProF's invariants unify data across scales and geometries, Figure **2**(c).

4. Discussion

The combined theoretical and experimental evidence shows that flow-rate dependence is an intrinsic property of steady-state two-phase flow in porous media. The DeProF framework is based on mechanistic – stochastic hierarchical upscaling. It provides a new, true-to-mechanism, phenomenological law, pertaining to steady-state, fully developed flows in porous media, resolving the disadvantages of the classical assumption of rate-independent relative permeability, especially so where intermittent flow of disconnected phase is substantial.

The verified scaling laws and invariants pave the way for rational upscaling and optimization of displacement processes in energy and environmental applications.

5. Conclusions

The DeProF theory represents a significant conceptual advance in modeling two-phase flow in porous media, especially so as long as fully-developed conditions are considered. It unifies pore-scale mechanics, statistical mechanics, and macroscopic transport laws under a coherent framework.

Experimental studies from 2016 to 2025 confirm the flow-dependent, universal scaling predicted by the theory, validating DeProF as a reliable basis for next-generation petrophysical characterization and enhanced recovery strategies.

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