Numerical and Geometric Optimization Techniques for Environmental Prediction Systems

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Introduction

Numerical environmental modeling is keeping the last decades a primary role in research and technological advances. The application of atmospheric and wave model outputs for renewable energy estimation and monitoring is particularly highlighted under the concerns posed by the recent economic crisis and the questions for global warming and climate change. Within this framework the utilization of optimization techniques which, in conjunction with mesoscale and regional wind/wave modeling systems, provide accurate environmental predictions in long and short term horizons is receiving increased attention.

In the present work, novel techniques for the estimation of the biases and uncertainty of numerical weather prediction systems are proposed based on the combination of dynamical statistical tools (Kalman filters) and recent advances in a relatively new branch of mathematics the Information Geometry. The latter implements techniques from the non-Euclidean geometry in statistics, targeting to the optimization of the solution of nonlinear problems. More precisely, the probability distributions obtained by simulated wind/wave data and the corresponding observations are categorized as elements of statistical manifolds, the appropriate geometric framework is clarified – avoiding classical simplifications associated with least square methods, and the discrepancies between the modeled and recorded datasets are measured by means of corresponding minimum length curves (geodesics). The latter are reached as solutions of second order differential equations for the study of which numerical techniques are employed. The proposed methodology is applied to selected areas of Greece targeting to the optimal estimation and monitoring of renewable energy sources.

Numerical Modeling for Wind/Wave Parameters

The validity for high quality wind/wave simulations is of critical importance today for a number of important applications:

✓ Global Warming	✓ Marine pollution
✓ Renewable energy estimation, monitoring and forecasting	✓ Ship safety
✓ Transportation	✓ Agricultural activities

The use of ***numerical prediction models***, in combination with available observations, has been recognized by the research and technical community as the main tool towards accurate environmental simulations/forecasts. Such models solve the main equations governing the atmosphere and wave evolution based on arithmetic schemes (finite differences on grid points or others).

The following models are utilized by our group for atmospheric and wave simulations:

Wave Model *WAM*

WAM - ECMWF parallel version (Komen et al., 1994; WAMDI group, 1988; Bidlot J. and Janssen P. 2003) is a third generation wave model, which computes spectra of random short-crested wind-generated waves.

- The model describes the evolution of a two-dimensional ocean wave spectrum.
- In contrast to first and second generation models, WAM introduces no ad hoc assumptions on the spectral shape.
- It computes the 2-d wave variance spectrum through integration of the transport equation

$$\frac{dF}{dt} + \frac{\partial}{\partial \phi}(\dot{\phi}F) + \frac{\partial}{\partial \lambda}(\dot{\lambda}F) + \frac{\partial}{\partial \theta}(\dot{\theta}F) = S,$$

  F represents the spectral density with respect to (f,θ,φ,λ), f denotes frequency, θ directions, φ latitudes, λ longitudes
- The source function S is represented as a superposition of the wind input *Sin*, white capping dissipation *Sdis*, and nonlinear transfer *Snl*

Atmospheric Model SKIRON

SKIRON has been developed at the University of Athens by the Atmospheric Modeling and Weather Forecasting Group based on the Eta/NCEP model

- It consists of various modules for pre- and post- processing together with a version of the Eta model appropriately coded in order to run on any parallel computer platform
- Is a full physics non-hydrostatic model with sophisticated convective, turbulence and surface energy budget scheme

Atmospheric Model RAMS

RAMS is a highly versatile numerical code, developed at Colorado State University and Mission Research Inc/ASTeR Division. It is considered as one of the most advanced modeling systems available today.

It is a merger of a non-hydrostatic cloud model and a hydrostatic mesoscale model. It is able to simulate atmospheric phenomena with resolution ranging from tens of kilometers to a few meters.

Models Limitations

Numerical wave (and atmospheric) models have been proved successful for the simulation of the general sea state conditions on global or intermediate scale.

However, when focusing on local characteristics systematic errors may appear due to:

- the strong dependence on the initial/boundary conditions,
- the inability to capture sub-scale phenomena
- the parameterization of certain atmospheric/wave procedures
- the lack of a dense observation network which could help on the systematic correction of initial conditions.

Ways out – Optimizing the Numerical Models Outputs

Increase the model resolution: It remains an open question if this leads to a considerable improvement of the forecast skill. Even if this is true, it also results to increased computational cost.

Assimilation systems.

- Used for correcting the initial conditions based on available observations
- Problems: Limited available/quality controlled observations over oceans, Limited spatial and temporal impact

Statistical post-processing methods for local adaptation

MOS methods, Neural networks:, Kalman filters

In all the above cases ***a cost function should be minimized***

For example, in the case of Kalman filters :

The evolution in time of an unknown process x_t *is described by the system equation:* $x_t = F_t{:}x_{t-1} + w_t$

A known process y_t *is used in connection with* x_t *by the observation equation* $y_t = H_t{:}x_t + v_t$

The filter is based on the minimization of the covariance matrix $E(x_t x_t^T)$ *of* x_t

Is this distance adequately measured ?

A serious simplification is made here: The distance/cost-function is measured by means of classical Euclidean Geometry tools.

Information Geometry

Information geometry is a relatively new branch of Mathematics applying methods and techniques of non-Euclidean geometry to stochastic processes.

- A main subject: Given two probability distributions or two data sets, is it possible to define a notion of ***distance*** between them?
- Families of probability distributions are recognized as (statistical) manifolds on which geometrical entities such as Riemannian metrics, can be naturally introduced.
- The geometrical framework in such a manifold is given by the ***Fisher information matrix*** with elements

$$g_{ij}(\xi) = \int \partial_i \ell(x;\xi)\partial_j \ell(x;\xi) p(x;\xi)dx$$

where $\ell(x;\xi) = log[p(x;\xi)]$ and p the distributions

- The minimum distance between two elements f_1 *and* f_2 of a statistical manifold is defined by the corresponding ***geodesic*** - the minimum length curve ω that connects them. Such a curve $\omega = (\omega_i): \mathbb{R} \to S$ is the solution of a system of 2nd order differential equations:

$$\omega_i''(t) + \sum_{j,k=1}^{n} \Gamma_{jk}^i(t)\,\omega'_j(t)\omega'_k(t) = 0, \qquad i = 1, 2, \ldots, n.$$

under the conditions $\omega(0) = f_1,\ \omega(1) = f_2$, where (Γ_{jk}^i) are the Christoffel symbols of the Levi-Civita connection with respect to the Fisher metric

Applications of Information Geometry to Wind/Wave Modeling

Information Geometric techniques can significantly support the optimization of the environmental predictions either via assimilation or post process systems.

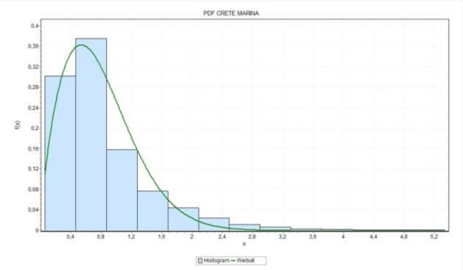
The main steps that should be taken are:

- Estimate the statistical distributions followed by the data in study.
- Establish the corresponding geometric environment, i.e. the appropriate statistical manifold.
- Use this framework in order to accurately estimate the distance between data sets and, therefore, adequately define the cost-functions used.

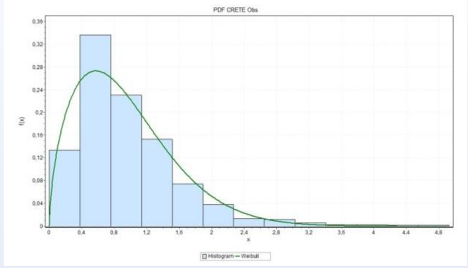
A test case for area of Greece

- The numerical models SKIRON and WAM were used to numerically wind speed and significant wave height values for a 10-year period (2001 – 2012) at a high spatial (5Km) and temporal resolution mode over different areas of Greece.
- The results were compared with corresponding records from satellites.

The testing area



The modeled data



The corresponding observations

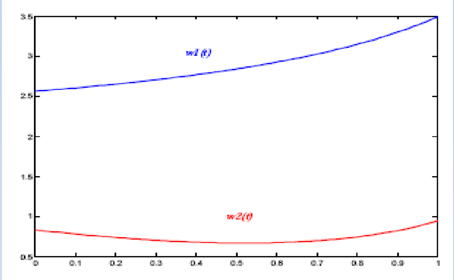
- Based on different statistical tests (Kolmogorov-Smirnov, Anderson-Darling) it was proved that the 2-parameter Weibull distribution fits well to the data in study, both modeled and observed data, but with different shape and scale parameters *(α, β)* .
- The obtained pdfs could be recognized as elements of the Weibull statistical manifold:

$$S = \left\{ f(x) = \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1} e^{-\left(\frac{x}{\beta}\right)^{\alpha}},\ \alpha,\beta > 0 \right\}$$

- The Fisher information matrix takes the form $G(\alpha,\beta) = \begin{bmatrix} \alpha^2\beta^2 & \beta(1-\gamma) \\ \beta(1-\gamma) & \frac{6(\gamma-1)^2+\pi^2}{6\alpha^2} \end{bmatrix} = \begin{bmatrix} 5.76 & 0.63 \\ 0.63 & 0.71 \end{bmatrix}$, where γ is the Euler Gamma.
- The corresponding geodesics, necessary to estimate distances between different data sets are solutions of the second order system:

$$\omega_1'' - 0.94(\omega_1')^2 + 1.39\omega_1'\omega_2' - 0.17(\omega_2')^2 = 0$$

$$\omega_2'' - 0.18(\omega_1')^2 + 0.55\omega_1'\omega_2' - 0.69(\omega_2')^2 = 0$$



- Numerical techniques should be utilized for solving such type of equations

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