

Benchmarking two simulation models for underwater and atmospheric sound propagation

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Abstract

Computational wave propagation models are widely used in underwater and atmospheric sound propagation simulation. In most realistic cases the physical domains involved are irregular. We have developed finite element techniques, applied to general irregular meshes and coupled with discrete, artificial absorbing boundary conditions of nonlocal type, for the Helmholtz equation and its ‘standard’ parabolic approximation. The physical domain is axially symmetric, with several fluid layers of variable acoustic properties. Boundaries and interfaces of general topography are allowed. The resulting models are referred to as the FENL and CNP1-NL models, respectively. We present results of the FENL model for underwater acoustic applications related to object identification and of the CNP1-NL for atmospheric sound propagation over an irregular terrain. © 2005 Elsevier Ltd. All rights reserved.

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1. Introduction

Computational wave propagation has lately received increased interest due to its applicability in various scientific fields, such as the simulation of sound waves propagation in the atmosphere and in the sea. The simulation of atmospheric and underwater sound propagation is widely used in problems in environmental sciences. To develop effective simulations, methods that provide accurate and low-cost computations are sought.

The direct problem of underwater sound propagation modeling is related mostly to industrial and military applications. However, object detection and identification techniques, based on the assessment of the distortion of the wave pattern, apply to the recognition of localized, abrupt changes in the acoustic properties due to the existence of currents and eddies.

Nonetheless, the direct problem of underwater sound propagation is implicitly related to environmental problems due to inverse problem applications. As an example, many methods that simplify and accelerate sea circulation models have as a prerequisite the computation of flow velocities and temperatures via inverse acoustic problems. On the other hand, atmospheric sound propagation modeling is directly related to problems in environmental sciences, like the control of ambient noise emitted, for example, from various man-made sources, like wind parks, airports and highways.

In this paper we consider two-dimensional sound propagation, by assuming a physical domain in cylindrical coordinates with axial symmetry. The domain consists of several fluid layers with variable acoustic properties. The interfaces and the sea bottom or the ground surface may have variable topography. In the following sections we present direct numerical solution methods, for the Helmholtz equation and its ‘standard’ parabolic approximation, based on finite element computations over general irregular meshes. We truncate the unbounded physical domains introducing an artificial

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boundary where we pose an exact, nonlocal, absorbing condition. Such a condition does not generate spurious reflections from the artificial boundary and reduces significantly the computational cost by avoiding the use of artificial absorbing layers, especially when wavelength size requires fine discretizations.

We consider the Helmholtz equation in cylindrical coordinates, in an axially symmetric domain,

$$\Delta p + k^2(z, r)p = 0, \quad (1)$$

where $p(z, r)$ is the acoustic pressure field due to a harmonic source of frequency f , $k(z, r) = k_0 n(z, r)$, $k_0 = 2\pi f / c_0$ is a reference wave number associated with the reference sound speed c_0 , and $n(z, r)$ is the index of refraction of the fluid medium. Here r denotes the horizontal distance (usually starting from the source) and z denotes the respective vertical distance. If the propagation has a limited aperture about the horizontal direction, with weak backscattering, and we are many wavelengths away from the source, we can assume that $p = u(z, r)v(r)$, where v satisfies $v_r = ik_0 v$ (Tappert, 1977). Then $u(z, r)$ satisfies the evolution equation (with r a time-like variable)

$$2ik_0 u_r + u_{zz} + k_0^2(n^2(z, r) - 1)u = 0, \quad (2)$$

known as the ‘standard’ parabolic approximation of the Helmholtz equation or the ‘standard’ parabolic equation. Eq. (2) is valid for propagation with aperture up to 15° . Variations of Eq. (2), the so-called ‘wide’ angle parabolic equations, deal with simulation of propagations with larger apertures (Lee et al., 2000).

In Section 2 we present the FENL model (Kampanis and Dougalis, 1999; Mitsoudis et al., 2002). FENL employs Eq. (1) in a suitable boundary value problem to simulate sound propagation in the sea. The physical domain is a waveguide, unbounded in the outflow (r) direction, with a bottom of variable bathymetry. A piecewise linear finite element solution is computed on an irregular mesh, suitably adjusted to a nonlocal transparent boundary condition along a vertical boundary in the outflow region.

In Section 3 we present the CNP1-NL model, which employs Eq. (2) in a suitable initial-boundary value problem to simulate atmospheric sound propagation over irregular ground. In the z direction the physical domain has an irregular lower boundary and is unbounded above. A nonlocal boundary condition is assumed along an artificial, horizontal upper boundary, accounting for the effect of the above atmospheric layer. A piecewise linear finite element solution in a suitable curvilinear coordinate system, is marched in range using second order centered finite differences. The curvilinear system is defined via a transformation of coordinates that simplifies the coupling with the nonlocal upper boundary condition.

In Section 4 we present results from the application of FENL and CNP1-NL models to underwater and atmospheric acoustic problems. The FENL model is used to examine the distortion of the sound field in the sea from the presence of axisymmetric discs (ring type structures) of different radii

within the domain of propagation. The CNP1-NL model is applied to the simulation of sound propagation over an irregular terrain where the nonlocal boundary condition accounts for an upper atmospheric layer with constant index of refraction. A preliminary result where $n^2(z, r)$ varies linearly in the upper layer is also shown. Comparisons with the approximate full-field solution provided by OASES (Schmidt, 1997) validate the results.

A brief conclusion is found in Section 5.

2. The FENL model

We consider Eq. (1) in an axially symmetric waveguide, unbounded in the outflow (r) direction, consisting of several fluid layers, separated by variable interfaces over a bottom of irregular bathymetry. Away from a harmonic source of frequency f (Hz) (assumed to be located at depth $z = z_s$, and range $r = 0$) the acoustic pressure $p(z, r)$ is determined by the p.d.e. (Eq. (1)) supplemented with a source boundary condition for small r , an outgoing radiation condition as r increases, a pressure-release surface condition $p(0, r) = 0$, a homogeneous Dirichlet or Neumann bottom condition $p = 0$ or $\partial p / \partial \nu = 0$ (here $\partial / \partial \nu$ denotes the normal derivative operator) and, in general, with the usual transmission conditions across layer interfaces (Kampanis and Dougalis, 1999).

In order to apply the finite element method, a finite computational domain is defined by the introduction of an artificial outflow boundary, where we pose the nonlocal, transparent boundary condition

$$\frac{\partial p}{\partial r} = T(p), \quad (3)$$

where T is the integral operator associated with the DtN map of the exterior field (Goldstein, 1982; Kampanis and Dougalis, 1999) evaluated on the artificial boundary. The operator T is expressed by an infinite sum involving the eigenvalues and eigenfunctions of the eigenvalue problem along z resulting from the separation of variables in Eq. (1).

The boundary value problem is discretized using a standard Galerkin formulation with linear finite elements on a general mesh that properly fits the interfaces. The nonlocal condition (Eq. (3)) is a generalized natural boundary condition and is approximated by a finite sum including only those eigenfunctions that contribute significantly. The computational needs for the solution of the discrete problem involve the following tasks:

1. Construct the finite element mesh T_h of the computational domain.
2. Assemble the global finite element matrices.
3. Construct the DtN operator T .
 - a. Define the 1D finite element mesh, imposed by T_h , on the artificial boundary.
 - b. Assemble the matrices for the eigenvalue problem.
 - c. Solve the resulting generalized eigenvalue problem.

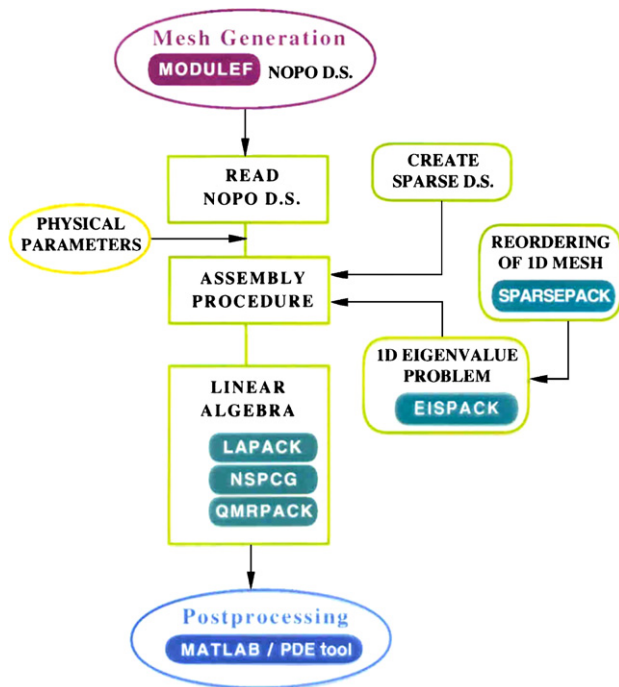


Fig. 1. Flowchart and interfaces of the FENL code.

4. Solve the indefinite linear system resulting from the finite element discretization of the b.v.p. for Eq. (1).
5. Perform the post-processing of the finite element solution.

These tasks are described in detail in Kampanis and Dougalis (1999) where the FENL code is presented. A schematic diagram of the structure of the FENL code is shown in Fig. 1. The FENL code is interfaced with freely available numerical software and a graphical tool to handle some of the above-mentioned tasks.

The FENL model provides accurate simulations of sound propagation and backscattering at any angle. It has been tested on several characteristic problems of sound propagation in various marine environments (Kampanis and Dougalis, 1999; Dougalis et al., 1998; Kampanis and Ekaterinaris, 2001; Kampanis and Flouri, 2003). The results were consistent with those from other standard codes in the literature, as well as with exact solutions where available. The memory requirements of the FENL model increase with frequency since finer finite element meshes are required. Therefore, FENL is most useful for low to medium frequency propagations up to medium ranges in shallow water.

The basic finite element code FENL can be found at the Ocean Acoustics Library at <http://www.hlsresearch.com/oalib/other/fenl>.

3. The CNP1-NL model

We consider Eq. (2) in an axially symmetric physical domain, with an irregular lower boundary corresponding to the

ground surface, and unbounded above (in the z direction). The physical domain may consist of several fluid layers, separated by horizontal interfaces. The p.d.e. (Eq. (2)) is supplemented with an outgoing radiation condition as z increases, a locally reacting boundary condition of natural (Neumann) type given by

$$u_z + \alpha(r)u_r + \beta(z, r)u = 0, \quad (4)$$

(here α and β depend, as functions of r , on the ground surface variability and roughness, cf. Kampanis (2002) for the exact formulae), a starting field at $r=0$, and the usual transmission conditions across layer interfaces.

A finite computational domain is defined by the introduction of an artificial horizontal boundary at a height z_T , above which the atmosphere has a constant index of refraction n_a . We pose a nonlocal, transparent boundary condition (Kampanis, 2002) in the form of the NtD map of the outer acoustic field, given by

$$u(z_T, r) = C \int_0^r \frac{1}{\sqrt{s}} e^{\frac{ik_a}{2}(n_a^2-1)s} u_z(z_T, r-s) ds, \quad (5)$$

where C is a suitable constant.

Marching in range a finite element solution, satisfying the boundary and interface conditions, could easily deal with the initial-boundary value problem at hand, under the assumption that the lower and upper boundaries of the computational domain are horizontal (Dougalis and Kampanis, 1996). In order to treat the irregular ground surface, we employ a curvilinear coordinate system (in range and height) fitting the irregular lower boundary. Note that the transform must conserve the form of Eq. (5), since it is convenient to implement in the discretization process. Therefore, a special transformation of coordinates, which reduces to identity within an artificial layer lying below the artificial upper boundary, is used to define the curvilinear system (Kampanis, 2002). (We note that the discrete analog of Eq. (5) is of the constrained (Dirichlet) type.) Even though this transform leads to a slightly more complicated p.d.e. and boundary condition than Eqs. (2) and (4), the transformed computational domain is now rectangular. Hence to advance the solution in range (i.e. for every r) the same finite element grid in z may be used. The finite element solution is a piecewise linear function, subjected to the resulting boundary conditions and is marched in range by a second order, implicit finite difference scheme.

The transformation of coordinates technique was first introduced and successfully operated in underwater acoustic propagation modeled by parabolic equations (Dougalis and Kampanis, 1996). With a suitable modification, it was applied successfully to the 'standard' parabolic equation, modeling sound propagation over an irregular terrain (cf. Kampanis and Ekaterinaris, 2001; Kampanis, 2002) where the CNP1-NL model has been initially presented. Since in practical applications, realistic ground surface topographies are required, a GIS interface for the CNP1-NL code has also been incorporated (Kampanis and Flouri, 2003).

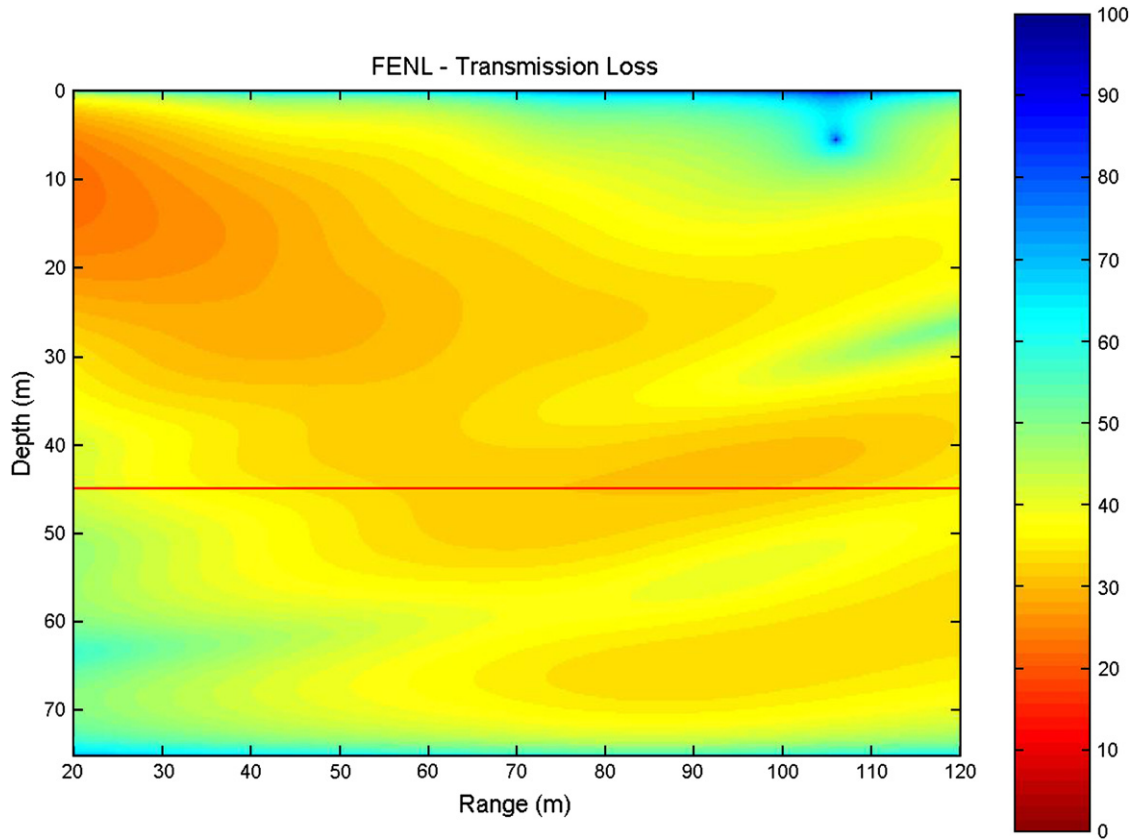


Fig. 2. FENL test case: undistorted transmission loss.

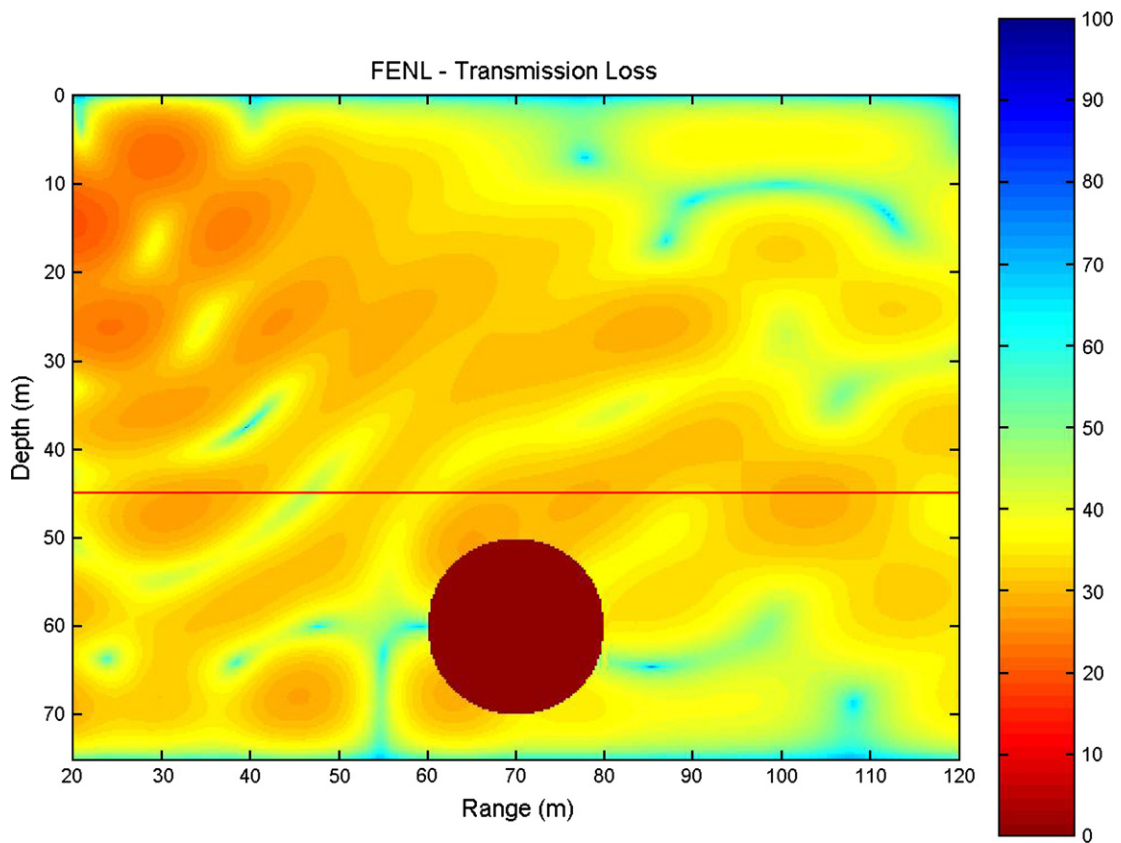


Fig. 3. FENL test case: distortion of the transmission loss by a rigid disc of diameter 20 m.

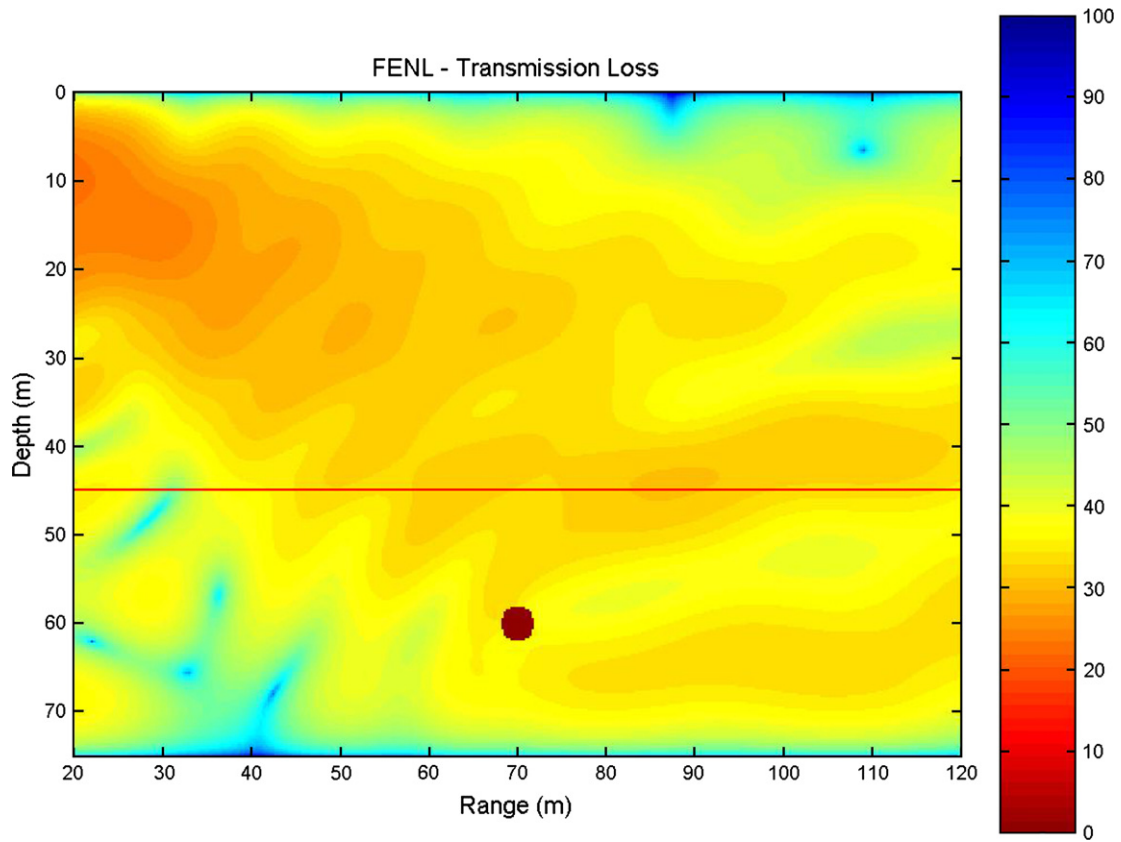


Fig. 4. FENL test case: distortion of the transmission loss by a rigid disc of diameter 4 m.

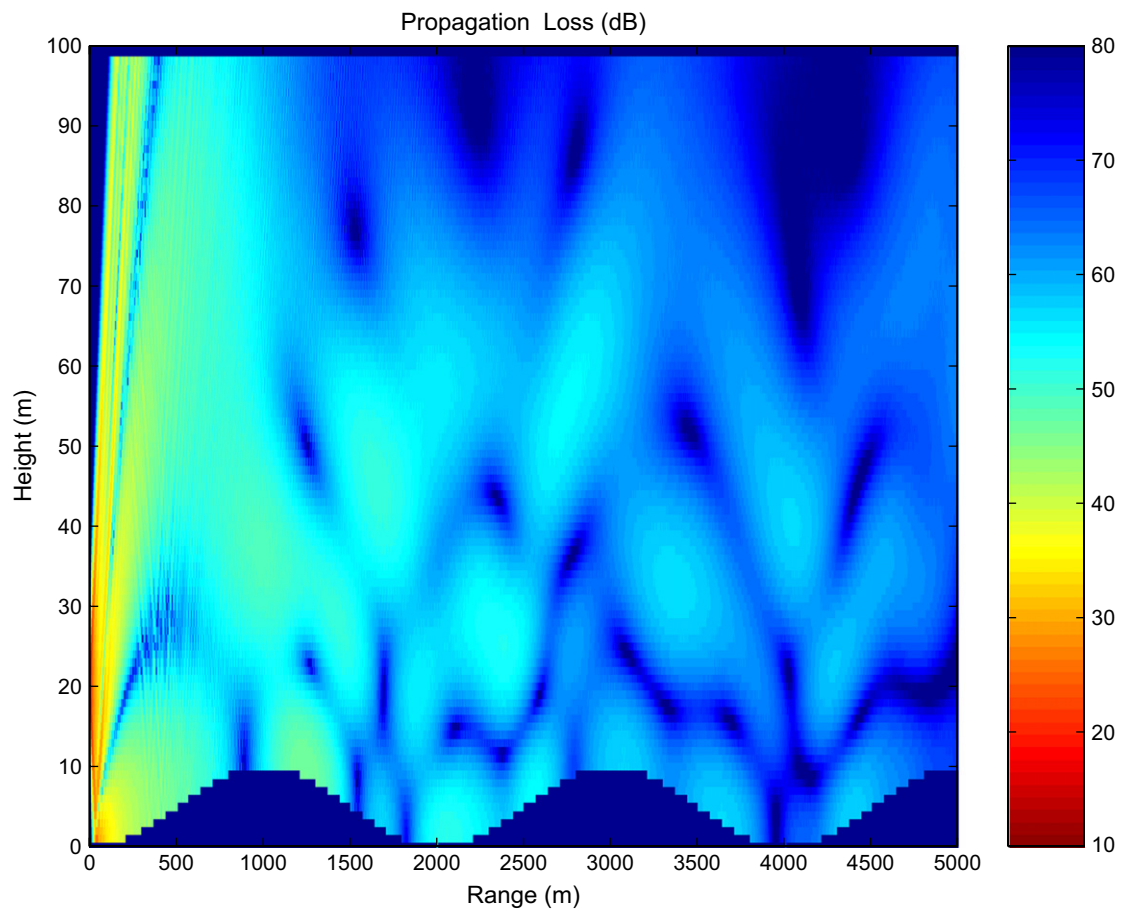


Fig. 5. CNP1-NL test case: sound field over an irregular terrain. Downward refracting atmosphere in computational domain. Nonlocal b.c. for constant n .

4. Examples

4.1. FENL application to the detection of a submerged rigid disc

The FENL code has been compared extensively with other standard codes for computational underwater acoustics in various underwater environments and showed an excellent agreement (Kampanis and Dougalis, 1999; Dougalis et al., 1998; Athanassoulis et al., in press).

In the test case considered here we take a rectangular domain of depth 75 m, consisting of two fluid layers, separated by a horizontal interface at 45 m and having densities of 1.0 and 1.2 g/cm³, and sound speeds 1500 and 1700 m/s, respectively. We consider propagation at a frequency of 75 Hz, due to a harmonic source (modeled by a Gaussian distribution of pressure values at a vertical, left boundary at a distance of 20 m from the source) (Mitsoudis et al., 2002; Dougalis et al., 2003).

Fig. 2 shows the undistorted transmission loss, i.e. the acoustic pressure field in a suitable logarithmic scale. Subsequent Figs. 3 and 4 show the distortions brought upon by the presence of rigid discs having diameters of 20 and 4 m,

respectively. Comparing Figs. 2 and 4 we may deduce that FENL is sensitive even to quite small obstacles.

4.2. CNP1-NL application to propagation over an irregular terrain

So far the CNP1-NL code has been applied with success, as is verified by comparisons with the OASES code, to the simulation of sound propagation over irregular terrain (Kampanis, 2002; Kampanis and Ekaterinaris, 2001; Kampanis and Flouri, 2003). A general refracting atmosphere has been considered within the computational domain. In the above atmospheric layer, whose effects are accounted via the nonlocal boundary condition (Eq. (5)), a constant index of refraction was assumed.

Fig. 5 shows the numerical results obtained by CNP1-NL for a sound field produced by a harmonic source of frequency 40 Hz, in a downward refracting atmosphere with sound speed $330 + 0.12z$ m/s, up to a height of $z = 100$ m, and constant speed of 342 m/s for larger z . The nonlocal condition (Eq. (5)) is applied to an artificial horizontal boundary placed at a height of $z_T = 100$ m.

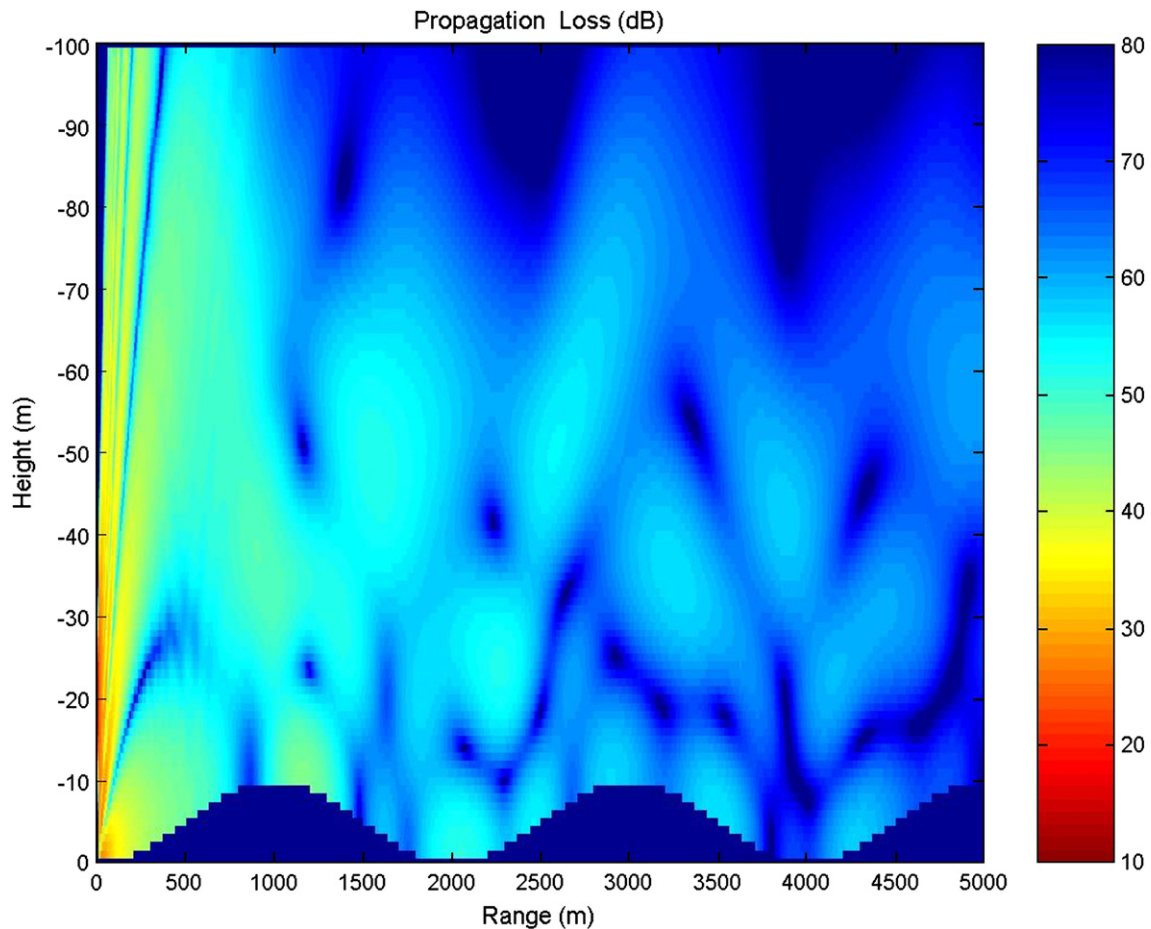


Fig. 6. CNP1-NL test case: sound field over irregular terrain. Downward refracting atmosphere in computational domain. Nonlocal b.c. for linear n^2 in upward refracting outer atmosphere.

For an atmospheric layer with linear n^2 the transparent boundary condition of nonlocal type as described in Dawson et al. (submitted for publication) has also been implemented in the CNP1-NL code. Fig. 6 shows a preliminary testing of the code for an upper atmospheric layer with $n^2(z) = 1 + 0.00075(z - 100)$, which corresponds to an upward refracting sound speed. This case involves propagation at limited apertures, since high-angle energy is driven upwards, and therefore is effectively modeled by the ‘standard’ parabolic equation. This is actually the reason that Figs. 5 and 6 show a quite similar wave pattern, since the most significant contribution to the propagation comes from the lower part of the atmosphere.

5. Conclusion

We have presented the FENL and CNP1-NL models, based on finite element discretizations coupled with nonlocal boundary conditions, for the Helmholtz equation and its ‘standard’ parabolic approximation, respectively, for sound propagation in refractive fluid media. The advantages of both codes are that they can handle irregular computational domains, with variable acoustic properties, and accommodate outgoing radiation conditions via suitable discretizations of the associated transparent boundary conditions of nonlocal type on an artificial boundary. We have also quoted some results from the successful benchmarking of these codes with underwater and atmospheric sound propagation applications.

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